Accepted Manuscript

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PII: S1389-1286(15)00320-5
DOI: 10.1016/j.comnet.2015.09.010
Reference: COMPNW 5683

To appear in: Computer Networks

Received date: 12 March 2015
Revised date: 25 July 2015
Accepted date: 14 September 2015


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Congestion Probabilities of Elastic and Adaptive Calls in Erlang-Engset Multirate Loss Models under the
Threshold and Bandwidth Reservation Policies

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Abstract

In this paper, we consider a single link of fixed capacity that accommodates calls of different service-classes with different bandwidth-per-call requirements. The link behaves as a multirate loss system. Calls of each service-class arrive in the link according to a Poisson (random) or a quasi-random process and have an exponentially distributed service time. Poisson or quasi-random arriving calls are generated by an infinite or finite number of traffic sources, respectively. Service-classes are also distinguished according to the behaviour of in-service calls, in elastic and adaptive service-classes. Elastic calls can compress their bandwidth by simultaneously increasing their service time. Adaptive calls tolerate bandwidth compression without affecting their service time. All calls compete for the available link bandwidth under the combination of the Threshold (TH) and the Bandwidth Reservation (BR) policies. The TH policy can provide different QoS among service-classes by limiting the number of calls of a service-class up to a predefined threshold, which can be different for each service-class. The BR policy reserves part of the available link bandwidth to benefit calls of high bandwidth requirements. The proposed models, for random or quasi-random traffic, do not have a product form solution for the determination of the steady state probabilities. However, we approximate both models by reversible Markov chains, and prove recursive formulas for the efficient calculation of the call-level
performance metrics, such as time and call congestion probabilities as well as link utilization. The accuracy of the proposed formulas is verified through simulation and found to be quite satisfactory.

**Keywords:** Poisson process; quasi-random; elastic-adaptive calls; threshold policy; reservation; Markov chain.

1. Introduction

Multirate loss models based on recursive formulas provide an efficient way for the call-level QoS assessment in modern communication networks, which accommodate elastic and adaptive traffic to a great extent. The term *multirate* expresses the fact that the traffic of the modeled system can be distinguished into several service-classes, according to the required bandwidth per call. Elastic traffic refers to in-service calls whose bandwidth can be compressed, while at the same time their service time increases (so that the product service time by bandwidth remains constant). Adaptive traffic (e.g., adaptive video) refers to in-service calls whose service time does not alter in the case of bandwidth compression. The call-level analysis of a single link that behaves as a pure loss system and accommodates elastic or adaptive calls of different service-classes is based on the classical Erlang Multirate Loss Model (EMLM) ([1], [2]).

In the EMLM, a link of capacity $C$ bandwidth units (b.u.) accommodates Poisson arriving calls of $K$ different service-classes. Calls of service-class $k$ ($k=1,\ldots,K$) have a peak-bandwidth requirement of $b_k$ b.u. that is the maximum number of b.u. required by a call. New calls compete for the available link b.u. under the Complete Sharing (CS) policy, whereby calls are blocked and lost if their required b.u. are not available in the link. Accepted calls cannot compress their bandwidth and remain in the link for an arbitrarily distributed service time [1]. The steady-state probabilities in the EMLM have a Product Form Solution (PFS). The latter leads to an accurate calculation of Call Blocking Probabilities (CBP) via the classical Kaufman-Roberts recursive formula ([1], [2]). The existence of this formula has led to numerous extensions of the
EMLM in wired (e.g., [3]-[18]), wireless (e.g., [19]-[27]) and optical networks (e.g., [28]-[33]). Many studies (e.g., [34]-[36]) focus on the potential numerical stability problems that can appear by using the Kaufman-Roberts formula. Since such problems can be well overcome by normalizations, as it is clearly pointed out in [36], we adopt the Kaufman-Roberts formula as a springboard to our analysis. If the call arrival is quasi-random, i.e., calls come from finite sources [37], [38], then the EMLM is extended to the Engset Multirate Loss Model (EnMLM) [4]. We name this extension EnMLM, since it provides the same Time Congestion (TC) probabilities with the Engset formula when one service-class is considered.

The incorporation of elastic and adaptive traffic in the EMLM and EnMLM is proposed in [39] and [40], respectively. We name these models Elastic-Adaptive EMLM (EA-EMLM) and Elastic-Adaptive (EA-EnMLM), respectively. In both models, service-class \( k \) calls have peak and minimum bandwidth requirements of \( b_k \) and \( b'_k \) b.u., respectively. The latter is defined as a proportion of \( b_k \), which is common for all service-classes. A new elastic/adaptive service-class \( k \) call is accepted in the system with \( b_k \) b.u. if they are available in the link. Otherwise, the system accepts this call by compressing the peak-bandwidth requirement \( b_k \) together with the bandwidth of all in-service calls. Bandwidth compression of a service-class \( k \) call is permitted down to \( b'_{k,\text{min}} \). Call blocking occurs when the value of \( b'_{k,\text{min}} \) of a new call is still higher than the available bandwidth. Bandwidth expansion takes place when an in-service call, whose bandwidth is compressed, departs from the system. In that case, the remaining in-service calls expand their bandwidth in proportion to their peak-bandwidth.

In this paper, we initially consider a link that accommodates Poisson arriving calls of elastic service-classes and modify the admission mechanism to include the Threshold (TH) and the Bandwidth Reservation (BR) policies. We name the proposed model E-EMLM/TH-BR. Furthermore, we study the impact of both policies in the EA-EMLM and name the proposed model EA-EMLM/TH-BR. Afterwards, we consider that the link accommodates quasi-random arriving calls of elastic traffic and study the impact of both policies in the E-EnMLM.
Finally, we extend the proposed E-EnMLM/TH-BR to include the case of adaptive traffic (EA-EnMLM/TH-BR).

In the TH policy, the number $n_k$ of in-service calls of service-class $k$ should not exceed a pre-defined threshold, after the acceptance of a new service-class $k$ call. Otherwise, the call is blocked and lost even if enough bandwidth is available. The TH policy is significant in teletraffic engineering, since it analyzes a multirate access tree network which accommodates $K$ service-classes [37] and may differentiate service-classes in terms of CBP or revenue rates by a proper threshold selection (see e.g., [41], [42]). Applications of the TH policy are numerous (see e.g., [43]-[47]). In [43], [44], QoS differentiation between new and handoff calls of the same service-class accommodated in the same cell is achieved by applying the TH policy to new calls. In [45], the TH policy provides service differentiation and, based on a proper pricing scheme, achieves revenue optimization in a mobile cellular system. As it is stated, it is difficult in real systems to compute the optimal call admission policy because of the “curse of dimensionality”; therefore, admission policies of simple structure (e.g., the TH policy) are necessary in practice. In [46], the TH policy is applied in the EMLM (EMLM/TH) and a recursive formula that resembles the Kaufman-Roberts formula is proposed for an accurate CBP calculation. Recently, in [47] the authors extend the EMLM/TH in the case of quasi-random arrivals (EnMLM/TH) and study the co-existence of the TH policy with the BR policy (EMLM/TH-BR and EnMLM/TH-BR).

The BR policy is used to reserve bandwidth to benefit calls of high bandwidth requirements and is mainly used when CBP equalization is required among calls of different service-classes. The fact that the BR policy has been extensively applied in wired (e.g., [48]-[56]), wireless (e.g., [57]-[62]) and optical networks (e.g., [63]-[65]) evinces its importance in call admission control.

In the aforementioned papers ([41]-[65]), the bandwidth of in-service calls cannot be altered. To the best of our knowledge, this is the first paper that considers the combination of the TH and the BR policies in a system that accommodates elastic and/or adaptive calls either of random or quasi-random input.
Note that the application of only the BR policy in the EA-EMLM and the EA-EnMLM has been studied in [66] and [67], respectively. To model the system, we use the Markov chain method. However, due to the existence of the compression/expansion mechanism, the reversibility of the Markov chains is destroyed, and the steady state probabilities in the proposed models (E-EMLM/TH-BR, EA-EMLM/TH-BR, E-EnMLM/TH-BR and EA-EnMLM/TH-BR) do not have a PFS. Therefore, we resort to approximate Markov chains which provide recursive formulas for the efficient determination of the link occupancy distribution and, consequently, CBP and link utilization. The accuracy of the proposed formulas is verified through simulation and found to be quite satisfactory. On the other hand, the comparison of the proposed models with existing models (EA-EMLM, EA-EnMLM, EMLM/TH, EnMLM/TH, EMLM/TH-BR, EnMLM/TH-BR) shows the new models’ necessity, and their consistency over changes of their parameters (e.g., compression factor and offered traffic-load).

The remainder of this paper is as follows: Section 2, contains three subsections: a) In Subsection 2.1, we present the basic assumptions of the E-EMLM/TH and the call admission mechanism, i.e., we do not initially consider the BR policy; the bandwidth compression/expansion mechanism is described along with a tutorial example in Sub-subsection 2.1.1. b) In Subsection 2.2, we show how to determine the steady state distribution of the number of calls in the system, present the recursive formula for the link occupancy distribution and provide formulas for the various performance measures. c) In Subsection 2.3, we extend our model to include adaptive traffic (EA-EMLM/TH). In Section 3, we extend the EA-EMLM/TH to include the BR policy (EA-EMLM/TH-BR). In Section 4 and 5, we extend the models of Sections 2 and 3 to include the case of quasi-random traffic (E-EnMLM/TH-BR and EA-EnMLM/TH-BR). In Section 6, we provide numerical results whereby the new models are compared to the EA-EMLM, EA-EnMLM, EMLM/TH, EnMLM/TH, EMLM/TH-BR and EnMLM/TH-BR and evaluated through simulation results. We conclude in Section 7. Appendices A, B and C include the proofs of the theorems provided in Subsections 2.2, 2.3 and 4.2, respectively.
2. The proposed model for elastic random traffic (E-EMLM/TH)

2.1. The system model

Consider a link of capacity $C$ b.u. that accommodates elastic calls of $K$ service-classes under the TH policy. Service-class $k$, $(k = 1, \ldots, K)$ calls follow a Poisson process with arrival rate $\lambda_k$ and request $b_k$ b.u. (peak-bandwidth requirement). Bandwidth compression is introduced in the system by allowing the occupied link bandwidth $j$ to virtually exceed $C$ up to $T$ b.u., i.e., $j = 0, 1, \ldots, T$. Let $\mathbf{n} = (n_1, n_2, \ldots, n_K)$ be the vector of all in-service calls and $\mathbf{b} = (b_1, b_2, \ldots, b_K)$ the vector of peak-bandwidth requirements, then $j = \mathbf{n} \cdot \mathbf{b} = \sum_{k=1}^{K} n_k b_k$.

The decision to accept a new service-class $k$ call in the system is based on the following constraints:

a) The number of in-service calls of service-class $k$, $n_k$, together with the new call, should not exceed a pre-defined threshold parameter, $n_k^*$, i.e., $n_k + 1 \leq n_k^*$. Otherwise the call is blocked and lost. This constraint expresses the TH policy.

b) If constraint (a) is met then:

b1) if $j + b_k \leq C$, the call is accepted in the system with its peak-bandwidth requirement $b_k$ and remains in the system for an exponentially distributed service time with mean $\mu_k^{-1}$.

b2) if $T \geq j + b_k > C$ the call is accepted in the system by compressing its $b_k$ b.u. together with the assigned bandwidth of all in-service calls. After compression, all calls share the $C$ b.u. in proportion to their peak-bandwidth, while the link operates at its full capacity $C$. This is the processor-sharing discipline [68]. The compressed bandwidth of the new service-class $k$ call is:

$$b_k' = r b_k = \frac{C}{j + b_k} b_k$$

(1)

where, with an abuse of notation, $r \equiv r(n_k^+ \mathbf{b}) = r(\mathbf{n}) = \frac{C}{\mathbf{n} \cdot \mathbf{b} + b_k} = \frac{C}{j + b_k}$. To keep constant the product service time by bandwidth per call, the mean value of the service time of the new service-class $k$ call becomes:

$$\frac{1}{\mu_k} = \frac{b_k}{b_k \mu_k} = \frac{j + b_k}{C \mu_k}$$

(2)
The compressed bandwidth of all in-service calls changes to \( b'_i = \frac{C}{\mu_k} b_i \) for \( i = 1, \ldots, K \) and \( \sum_{i=1}^K n_i b'_i = C \). Similarly, their remaining service-time increases by a factor of: \( \frac{b_k + b'_k}{C} \). The minimum bandwidth given to a service-class \( k \) call is:

\[
b'_{k,\text{min}} = r_{\text{min}} b_k = \frac{C}{T} b_k
\]

where \( r_{\text{min}} = C/T \) and is common for all service-classes.

A new service-class \( k \) call, with \( b_k \) b.u., is blocked and lost if \( j + b_k > T \).

Note that increasing \( T \) decreases \( r_{\text{min}} \) and increases the delay of service-class \( k \) calls (compared to the initial service time \( \mu_k^{-1} \)). Thus, \( T \) can be chosen so that this delay remains within acceptable levels.

When an in-service call, with compressed bandwidth \( b'_i \), completes its service and departs from the system, then the remaining in-service calls expand their bandwidth to \( b''_i \) (\( i = 1, \ldots, K \)) in proportion to \( b_i \) as follows:

\[
b''_i = \min \left( b_i, b'_i + \frac{b'_i b_{k'}}{\sum_{k=1}^K n_k b_k} \right)
\]

2.1.1. Tutorial Example

The following tutorial example illustrates the compression/expansion mechanism. Let \( C = 4 \) b.u., \( T = 8 \) b.u., \( K = 2 \) service-classes, \( \lambda_1 = \lambda_2 = 1 \) calls/time unit, \( b_1 = 2 \) b.u., \( b_2 = 4 \) b.u and \( \mu_1^{-1} = \mu_2^{-1} = 1 \) time unit. The TH parameters are \( n_1^* = 3 \) and \( n_2^* = 1 \). This system has 7 states \( n = (n_1, n_2) \) presented in Table 1 together with \( j = n_1 b_1 + n_2 b_2 \), before and after bandwidth compression.

Consider now the cases of a call arrival/departure.

- **Call arrival.** A new 2\textsuperscript{nd} service-class call arrives in the system while the current state is \( (n_1, n_2) = (2, 0) \) and \( j = C = 4 \) b.u. Since \( j' = j + b_2 = T = 8 \) b.u., the call is accepted in the system after bandwidth compression has been applied to these three calls. The new state becomes \( (n_1, n_2) = (2, 1) \). In this state, calls of both service-classes compress their bandwidth to:

\[
b'_{1,\text{min}} = r(2, 1) b_1 = r_{\text{min}} b_1 = \frac{4}{8} b_1 = 1.0,
\]

\[
b'_{2,\text{min}} = r(2, 1) b_2 = r_{\text{min}} b_2 = \frac{4}{8} b_2 = 2.0
\]
so that \( j = n_1b_{1,\min} + n_2b_{2,\min} = 2 + 2 = 4 = C \). Similarly, \( \mu_1^{-1}, \mu_2^{-1} \) become \( \frac{\mu_1^{-1}}{C_{\min}} = \frac{\mu_2^{-1}}{C_{\min}} = 2.0 \) so that \( b_1\mu_1^{-1} \) and \( b_2\mu_2^{-1} \) remain constant.

• Call departure. Let the system be in state \((n_1, n_2) = (2, 1)\) when a 1st service-class call departs from the system. Then, its assigned bandwidth \( b'_{1,\min} = 1 \) is shared to the remaining two calls in proportion to their peak-bandwidth. Thus, in the new state \((n_1, n_2) = (1, 1)\) the 1st and 2nd service-class calls expand their bandwidth to \( b'_{1} = \frac{4}{5}b_1 = 1.333 \) b.u. and \( b'_{2} = \frac{4}{5}b_2 = 2.667 \) b.u., respectively. Thus, \( j = C = 4 \) b.u. Furthermore, the service times of both calls are decreased to \( \frac{6}{4}\mu_1^{-1} = 1.5 \) and \( \frac{6}{4}\mu_2^{-1} = 1.5 \) time units.

Figure 1 shows the state transition diagram of this example, together with \( n^*_1, n^*_2 \). If we apply the Kolmogorov’s criterion (flow clockwise = flow counterclockwise) [69] in the four adjacent states \((n_1, n_2) = (1, 0), (1, 1), (2, 1)\) and \((2, 0)\) then this criterion does not hold, i.e., the Markov chain is not reversible.

To circumvent this problem, we use state-dependent factors per service-class \( k, \phi_k(n) \), which have a similar role with \( r(n) \) and lead to a reversible Markov chain. The latter, facilitates the recursive calculation of the link occupancy distribution (see section 2.2). To ensure reversibility, \( \phi_k(n) \)'s have the form:

\[
\phi_k(n) = \begin{cases} 1 & \text{when } nb \leq C \text{ and } n \in \Omega \\ \frac{x(n)}{x(n-k)} & \text{when } C < nb \leq T \text{ and } n \in \Omega \\ 0 & \text{otherwise} \end{cases}
\]  

where \( \Omega \) is the system’s state space denoted by \( \Omega = \{n: 0 \leq nb \leq T, n_k \leq n^*_k, k = 1, ..., K\} \), \( nb = \sum_{k=1}^{K} n_k b_k \), \( n = (n_1, n_2, n_k-1, n_k, n_k+1, ..., n_K) \) and \( n^{-}_k = (n_1, n_k-1, n_{k+1}, ..., n_K) \).

In (6), \( x(n) \) is a state multiplier. To ensure that \( \sum_{k=1}^{K} n_k b_k \phi_k(n) = C \), whenever \( T \geq nb > C \), \( x(n) \) is defined as:

\[
x(n) = \begin{cases} 1 & \text{when } nb \leq C, n \in \Omega \\ \frac{1}{C} \sum_{k=1}^{K} n_k b_k x(n^{-}_k) & \text{when } C < nb \leq T, n \in \Omega \\ 0 & \text{otherwise} \end{cases}
\]
Note that $\phi_k(n)$’s are used to model state dependencies of transition rates and create reversible Markov chains (see e.g., [15]-[17], [19], [39]-[40]).

Figure 2 shows the modified state transition diagram due to $\phi_k(n)$’s. By considering again the states $(n_1, n_2) = (1, 0), (1, 1), (2, 1)$ and $(2, 0)$ we see that the Kolmogorov’s criterion holds, since:

$$\lambda_2 \lambda_1 \mu_2 \phi_2(2, 1) \mu_1 = \lambda_1 \lambda_2 \mu_1 \phi_1(2, 1) \mu_2 \phi_2(1, 1) \Rightarrow \phi_2(2, 1) = \phi_1(2, 1) \phi_2(1, 1) \Rightarrow x(2, 0) = x(1, 0) = 1.$$

In Table 2 we present the values of $r(n)$ (common for both service-classes), $\phi_1(n)$ and $\phi_2(n)$. We see that in state $(n_1, n_2) = (2, 1)$ the factors $\phi_1(2, 1)$, $\phi_2(2, 1)$ are different from $r(2, 1)$, a fact that leads to different values for the compressed bandwidth of calls (a 1st and a 2nd service-class call occupies $\phi_1(2, 1)b_1 = 1.2$ b.u. and $\phi_2(2, 1)b_2 = 1.6$ b.u., respectively).

2.2. The analytical model (E-EMLM/TH)

The following theorem provides a recursive formula for the calculation of the un-normalized values of the link occupancy distribution $G(j)$, in the proposed E-EMLM/TH.

Theorem 1

The un-normalized values of the link occupancy distribution, $G(j)$ are determined by the following recursive formula:

$$G(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{\min(C,j)} \sum_{k=1}^{K} a_k b_k [G(j-b_k) - T_k(j-b_k)], & \text{for } j=1,...,T \\
0, & \text{otherwise}
\end{cases}$$  (7)

The proof of Theorem 1 is provided in Appendix A.

The computational complexity of (7) is in the order of $O(K^m T)$, where $m = \max \{ \lfloor T/n_k b_k \rfloor : k = 1,...,K \}$. If the bandwidth compression mechanism is ignored, then we have the Tsang – Ross recursion (see A.6, in Appendix A) whose computational complexity is in the order of $O(K^m C)$, where $m = \max \{ \lfloor C/n_k b_k \rfloor : k = 1,...,K \}$[46].
Having determined \( G(j) \)'s we calculate the CBP of service-class \( k \), \( B_k \), as follows:

\[
B_k = \sum_{j=T-b_k+1}^{T} G^{-1} G(j) + \sum_{j=n^*_k b_k}^{T-b_k} G^{-1} T_k(j)
\]  

(8)

where \( G = \sum_{j=0}^{T} G(j) \) is the normalization constant.

Another important performance measure is the link utilization, \( U \), given by:

\[
U = \sum_{j=1}^{C} j G^{-1} G(j) + \sum_{j=C+1}^{T} C G^{-1} G(j)
\]  

(9)

In (7) and (8), the knowledge of \( T_k(j) \) is required. According to (A.7) of Appendix A, \( T_k(j) \) should be determined when \( j = n^*_k b_k, \ldots, T - b_k \). We consider two subsets: 1) \( n^*_k b_k \leq j \leq C \) (where no bandwidth compression takes place) and 2) \( C + 1 \leq j \leq T - b_k \) (where bandwidth compression exists).

For subset (1), consider a system of capacity \( F_k = C - n^*_k b_k \) that accommodates all service-classes but service-class \( k \) and define \( r_k(j), j=0, \ldots, F_k \), as:

\[
r_k(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{2} \sum_{i=1, i \neq k}^{K} a_i b_i \left[ r_k(j-b_i) - T_i(j-b_i) \right], & \text{for } j = 1, \ldots, F_k \\
0, & \text{otherwise}
\end{cases}
\]  

(10)

We compute the un-normalized values of \( T_k(j) \) as:

\[
T_k(j) = \frac{a_k^{n^*_k}}{n^*_k!} r_k(j-n^*_k b_k), \quad n^*_k b_k \leq j \leq C
\]  

(11)

In (11), the term \( a_k^{n^*_k}/n^*_k! \) is expected, since for \( n^*_k b_k \leq j \leq C \) the number of in-service calls of service-class \( k \) is \( n^*_k \).

For subset (2), \( T_k(j) \) is given by a similar to (A.4) formula (Appendix A):

\[
T_k(j) = \frac{a_k^{n^*_k}}{n^*_k!} \sum_{n \in \Omega_j} x(n) \prod_{i=1, i \neq k}^{K} \frac{a_i^{n_i}}{n_i!}
\]  

(12)

where:

\[
\Omega_j = \left\{ n \in \Omega : n^*_k b_k + \sum_{i=1, i \neq k}^{K} n_i b_i = j, \ C < j \leq T - b_k \right\}.
\]
In (12), \( T_k(j) \) is calculated only for a subset of \( \Omega \), defined by \( C < j \leq T - b_k \) and only under the assumption that \( n_k = n^*_k \). This means that enumeration of the subset of \( \Omega \) is needed for those states \( n = (n_1, n_2, ..., n_k = n^*_k, ..., n_K) \) where \( C < nb \leq T - b_k \). Assuming that \( T \) should not be chosen to be much higher than \( C \) (otherwise the increase of delay for elastic may be unacceptable for some applications) the subset of \( \Omega \) will not become large. In general, the computational complexity of (24) grows exponentially with \( K - 1 \) (since for service-class \( k \) we know that \( n_k = n^*_k \)) and can be in the order of \( O((T - b_k - C)^{(K-1)}) \). Assuming the existence of the CS policy and ignoring the bandwidth compression mechanism, the computational complexity becomes \( O(C^K) \) [1].

2.3. Extension of the E-EMLM/TH to include adaptive random traffic (EA-EMLM/TH)

Adaptive traffic is a variant of elastic traffic, since adaptive calls can tolerate bandwidth compression without altering their service time. To include adaptive traffic in the E-EMLM/TH, we assume that \( K_e \) and \( K_a \) are the number of elastic and adaptive service-classes, respectively. The single link accommodates \( K \) service-classes of Poisson arriving calls, where \( K = |K_e| + |K_a| \).

Consider again the tutorial example of section 2 and let calls of the 2nd service-class be adaptive. The state space and the occupied link bandwidth remain the same (see Table 1). Figure 3 shows the new state transition diagram. Compared to Fig. 1, the differences are in the values of \( \mu_2 \) which do not alter. If we consider the states \( (n_1, n_2): (1,0), (1,1), (2,1), (2,0) \), we see that Kolmogorov’s criterion does not hold and therefore the Markov chain is not reversible.

**Theorem 2** A reversible Markov chain is obtained by using the state-dependent factors per service-class \( k \), \( \phi_k(n) \), of (5) where \( x(n) \) is given by:

\[
x(n) = \begin{cases} 
1 & \text{if } nb \leq C, \quad n \in \Omega \\
\frac{1}{B} \left( \sum_{k \in K_e} n_k b_k x(n_k^*) + r(n) \sum_{k \in K_a} n_k b_k x(n_k^*) \right), & \text{if } C < n \leq T, \quad n \in \Omega \\
0 & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (13)
where $r(n) = C/nb$.

The proof of Theorem 2 is provided in Appendix B.

In Table 3 we present $r(n)$, $\phi_1(n)$ and $\phi_2(n)$. Compared to Table 2, we see that adaptive traffic modifies $\phi_1(n)$ and $\phi_2(n)$.

Based on (13) and (B.1)-(B.3) (of Appendix B) and following the analysis of section 2, it can be proved that $G(j)$’s are given by the following formula:

$$G(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{\min(C,j)} \sum_{k \in K_e} a_k b_k [G(j-b_k) - T_k(j-b_k)], & \text{for } j = 1, \ldots, T \\
\frac{1}{j} \sum_{k \in K_a} a_k b_k [G(j-b_k) - T_k(j-b_k)], & \text{for } j = 1, \ldots, T \\
0, & \text{otherwise}
\end{cases}$$  \hspace{1cm} (14)

In (14), the knowledge of $T_k(j)$ is required for $j = n_k^* b_k, \ldots, T - b_k$. We consider two subsets: 1) $n_k^* b_k \leq j \leq C$ and 2) $C + 1 \leq j \leq T - b_k$. For subset (1), we use (10) and (11), while for subset (2) we use (12) where the values of $x(n)$ are given by (13). In both subsets, $K = |K_e| + |K_a|$.

Having determined $G(j)$ we calculate the CBP of service-class $k$, $B_k$ and the link utilization, $U$, via (8) and (9), respectively.

3. The proposed EA-EMLM/TH including the BR policy (EA-EMLM/TH-BR)

Consider again a link of capacity $C$ b.u. that accommodates $K$ service-classes, where $K = |K_e| + |K_a|$ and $K_e, K_a$ are the number of elastic and adaptive service-classes, respectively. A new service-class $k (k = 1, \ldots, K)$ call has a peak-bandwidth requirement of $b_k$ b.u. and a BR parameter $t_k$ that expresses the reserved b.u. used to benefit calls of all other service-classes except from service-class $k$. If $j + b_k \leq T - t_k$ and the number of in-service calls of service-class $k$ in the steady state, $n_k$, plus the new call, does not exceed $n_k^*$, i.e., $n_k + 1 \leq n_k^*$ then the call is accepted in the link. Otherwise the call is blocked and lost.
To determine $G(j)$ in the EA-EMLM/TH-BR we propose the following approximate but recursive formula:

$$G(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{a}\sum_{k \in K} a_k D_k(j-b_k) [G(j-b_k) - T_k(j-b_k)] + \\
\frac{1}{j} \sum_{k \in K} a_k D_k(j-b_k) [G(j-b_k) - T_k(j-b_k)], & \text{for } j = 1, \ldots, T \\
0, & \text{otherwise} 
\end{cases}$$

(15)

where:

$$D_k(j-b_k) = \begin{cases} 
b_k & \text{for } j \leq T - t_k \\
0 & \text{for } j > T - t_k 
\end{cases}$$

(16)

A basic characteristic of the BR policy is that it ensures CBP equalization among different service-classes by a proper selection of the BR parameters. If, for example, CBP equalization is required between calls of two service-classes with $b_1 = 1$, $b_2 = 10$, respectively, then $t_1 = 9$ b.u., $t_2 = 0$ b.u. so that $b_1 + t_1 = b_2 + t_2$.

The application of the BR policy in the EA-EMLM/TH-BR is based on the assumption (approximation) that the population of service-class $k$ calls is negligible in states $j > T - t_k$ and is incorporated in (15) by the variable $D_k(j-b_k)$ given in (16). The states $j > T - t_k$ belong to the so-called reservation space. Note that the population of calls of service-class $k$ in the reservation space may not be negligible. In [49], [50] a complex procedure is implemented that takes into account this population in the EMLM and Engset multirate state-dependent loss models, respectively. However, according to [50] this procedure may not increase the accuracy of the CBP results compared to simulation.

Note at this point that the approximations introduced in the models by the $\phi_k(n)$’s and the $D_k(j-b_k)$’s factors are different in nature. The first approximation arises because of the necessity to analyze a model which has a reversible Markov chain, while the second approximation arises due to the introduction of the BR policy in a model.

If the link accommodates only elastic service-classes (E-EMLM/TH-BR)
then (15) becomes:

$$G(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{\min(C,j)} \sum_{k=1}^{K} a_k D_k(j-b_k) [G(j-b_k) - T_k(j-b_k)], & \text{for } j = 1, \ldots, T \\
0, & \text{otherwise} 
\end{cases}$$

(17)

Similarly to the EA-EMLM/TH, we determine the CBP of service-class $k$, $B_k$, based on two groups of states: i) those states where the available link bandwidth is less than $b_k + t_k$ b.u. when the new call arrives in the system; this happens when $T - b_k - t_k + 1 \leq j \leq T$ and ii) those states where the available link bandwidth is enough to accept the new call, i.e. $j \leq T - b_k - t_k$, but the number of in-service calls of service-class $k$ equals to $n^*_k$; the latter implies that $n^*_k b_k \leq j \leq T - b_k - t_k$. Thus, the values of $B_k$ are calculated by:

$$B_k = \sum_{j=T-b_k-t_k+1}^{T} G^{-1}(j) + \sum_{j=n^*_k b_k}^{T-b_k-t_k} G^{-1}T_k(j)$$

(18)

where $G = \sum_{j=0}^{C} G(j)$ is the normalization constant.

As far as the link utilization is concerned, it can be determined by (9).

In (15), (17) and (18) the knowledge of $T_k(j)$ is required for $n^*_k b_k \leq j \leq T - b_k - t_k$. We consider again two subsets: 1) $n^*_k b_k \leq j \leq C$ and 2) $C + 1 \leq j \leq T - b_k - t_k$. For subset (1), we use (10) and (11) where $F_k = T - b_k - t_k - n^*_k b_k$, while for subset (2) we use (12) where the values of $x(n)$ are given by (13) and

$$\Omega'_j = \left\{ n \in \Omega : n^*_k b_k + \sum_{i=1, i \neq k}^{K} n_i b_i = j, \ C < j \leq T - b_k - t_k \right\}.$$

4. The proposed model for elastic quasi-random traffic (E-EnMLM/TH)

4.1. The system model

Consider a link of capacity $C$ b.u. that accommodates elastic calls of $K$ service-classes. Service-class $k$ ($k = 1, \ldots, K$) calls come from a finite source population $N_k$ and have a peak-bandwidth requirement of $b_k$ b.u. The mean arrival rate of service-class $k$ idle sources is $\lambda_k = (N_k - n_k) v_k$, where $n_k$ is the
number of in-service calls and $v_k$ is the arrival rate per idle source. The offered traffic-load per idle service-class $k$ source is $a_{k,fin} = v_k/\mu_k$ (in erl), where $\mu_k$ is the mean value of the exponentially distributed service time of a service-class $k$ call. This is the quasi-random process [38]. If $N_k \to \infty$ for $k = 1, ..., K$ and the total offered traffic-load remains constant, then a Poisson process arises.

The introduction of bandwidth compression in the system and the call admission mechanism are similar to the E-EMLM/TH (see Subsection 2.1) and thus their description is omitted. As far as the tutorial example of Sub-subsection 2.1.1 is concerned the only difference is in the arrival process which is now quasi-random. Thus, Figs. 1 and 2 take now the form of Figs. 4 and 5, respectively. In addition, the values of $\phi_k(n)$’s and $x(n)$’s are given by (5) and (6), respectively.

4.2. The analytical model

The following theorem provides a recursive formula for the calculation of $G_{fin}(j)$’s in the proposed E-EnMLM/TH.

**Theorem 3**

The un-normalized values of the link occupancy distribution, $G_{fin}(j)$, satisfy

\[
G_{fin}(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{\min(c, j) \sum_{k=1}^{K} (N_k - n_k + 1)a_{k,fin}b_k[G_{fin}(j-b_k) - T_{k,fin}(j-b_k)]}, & \text{for } j = 1, ..., T \\
0, & \text{otherwise} 
\end{cases} 
\]  

(19)

The proof of Theorem 3 is provided in Appendix C.

Having determined $G_{fin}(j)$’s we calculate the TC probabilities of service-class $k$, $P_{bk}$, as:

\[
P_{bk} = \sum_{j=T-b_k+1}^{T} G^{-1}G_{fin}(j) + \sum_{j=n_kb_k}^{T-b_k} G^{-1}T_{k,fin}(j) 
\]

(20)

where $G = \sum_{j=0}^{T} G_{fin}(j)$ is the normalization constant.

Similarly, to determine the Call Congestion (CC) probabilities of service-class $k$, $B_k$, we use (20), where $G_{fin}(j)$’s and $T_{k,fin}(j)$’s are calculated for a
system with $N_k-1$ sources. TC probabilities refer to the proportion of time the system is congested while CC probabilities refer to CBP. In a Poisson arrival process, TC and CC probabilities coincide (PASTA property [38]).

To determine the link utilization, $U$, we can use the formula:

$$U = \sum_{j=1}^{C} jG^{-1}G_{fin}(j) + \sum_{j=C+1}^{T} CG^{-1}G_{fin}(j)$$

(21)

To calculate $T_{k,fin}(j)$, in (19) and (20) we consider two subsets: 1) $n_k^* b_k \leq j \leq C$ and 2) $C + 1 \leq j \leq T - b_k$.

For subset (1), we consider a system, of capacity $F_k = C - n_k^* b_k$, that accommodates all service-classes but $k$ and define $r_{k,fin}(j)$, $j = 0, ..., F_k$, as:

$$r_{k,fin}(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{j} \sum_{i=1, i \neq k}^{K} (N_i - n_i + 1) a_i,fin b_i [r_{k,fin}(j - b_i) - T_{i,fin}(j - b_i)], & \text{for } j = 1, ..., F_k \\
0, & \text{otherwise} 
\end{cases}$$

(22)

Next, we compute the un-normalized values of $T_{k,fin}(j)$ as:

$$T_{k,fin}(j) = \left( \begin{array}{c} N_k \\ n_k^* \end{array} \right) a_{k,fin} n_k^* r_{k,fin}(j - n_k^* b_k), \quad n_k^* b_k \leq j \leq C$$

(23)

In (23), the term $\left( \begin{array}{c} N_k \\ n_k^* \end{array} \right) a_{k,fin} n_k^*$ is expected, since for $n_k^* b_k \leq j \leq C$ the number of in-service calls of service-class $k$ is $n_k^*$.

For subset (2), $T_{k,fin}(j)$ is given by a similar to (C.4) formula (Appendix C):

$$T_{k,fin}(j) = \left( \begin{array}{c} N_k \\ n_k^* \end{array} \right) a_{k,fin} \sum_{n \in \Omega'} x(n) \prod_{i=1, i \neq k}^{K} \left( \begin{array}{c} N_i \\ n_i \end{array} \right) a_{i,fin}$$

(24)

where:

$$\Omega'_j = \left\{ n \in \Omega : n_k b_k + \sum_{i=1, i \neq k}^{K} n_i b_i = j, \quad C < j \leq T - b_k \right\}$$

To calculate $G_{fin}(j)$’s and $r_{k,fin}(j)$’s via (19) and (22), respectively, the knowledge of $n_k$ in each state $j$, $n_k(j)$, is required. We approximate $n_k(j)$ by
the mean number of service-class \( k \) calls in state \( j \), \( y_k(j) \), when calls follow a Poisson process, i.e., \( n_k(j) \approx y_k(j) \). This means that the values of \( y_k(j) \) will be determined via the E-EMLM/TH. Such approximations are common in the literature and induce little error (e.g., [11], [40], [47], [71]). Thus, to determine \( G_{f,i}(j) \)'s and \( r_{k,f,i}(j) \)'s we propose the formulas:

\[
G_{f,i}(j) = \begin{cases} 
  1, & \text{for } j = 0 \\
  \frac{1}{\min(c,j)} \sum_{k=1}^{K} (N_k - y_k(j-b_k)) a_{k,f,i} b_k \left[ G_{f,i}(j-b_k) - T_{k,f,i}(j-b_k) \right], & \text{for } j = 1, \ldots, T \\
  0, & \text{otherwise}
\end{cases}
\]  

(25)

where:

\[
y_k(j) G(j) = \frac{1}{\min(c,j)} a_k b_k \left[ G(j-b_k) - T_k(j-b_k) \right] (y_k(j-b_k) + 1) + \frac{1}{\min(c,j)} \sum_{i=1, i \neq k}^{K} a_i b_i \left[ G(j-b_i) - T_i(j-b_i) \right] y_k(j-b_i)
\]

(26)

\[
r_{k,f,i}(j) = \begin{cases} 
  1, & \text{for } j = 0 \\
  \frac{1}{j} \sum_{i=1, i \neq k}^{K} (N_i - y_{i,k}(j-b_i)) a_{i,f,i} b_i \left[ r_{k,f,i}(j-b_i) - T_{i,f,i}(j-b_i) \right], & \text{for } j = 1, \ldots, F_k \\
  0, & \text{otherwise}
\end{cases}
\]

(27)

where:

\[
y_{i,k}(j) r_{i,k}(j) = \frac{1}{\min(c,j)} a_k b_k \left[ r_{i,k}(j-b_i) - T_{i,k}(j-b_i) \right] (y_{i,k}(j-b_i) + 1) + \frac{1}{\min(c,j)} \sum_{x=1, x \neq i, k}^{K} a_x b_x \left[ r_x(j-b_x) - T_x(j-b_x) \right] y_{i,k}(j-b_x)
\]

(28)

The proof of (26) and (28) is similar to that of Theorem 2 in [39] and thus is omitted.

The following algorithm summarizes the procedure for the calculation of TC probabilities, CC probabilities and link utilization in the E-EnMLM/TH:

1) Determine \( G(j) \)'s, \( r_k(j) \)'s and \( T_k(j) \)'s of the corresponding Poisson model (E-EMLM/TH) via (7), (10) and (11)-(12), respectively.

2) Determine \( y_k(j) \) of the E-EMLM/TH via (26).
3) Determine $T_{k,\text{fin}}(j)$’s of the E-EnMLM/TH, via (23) when $n_k^* b_k \leq j \leq C$ and via (24) when $C + 1 \leq j \leq T - b_k$.

4) Determine $G_{\text{fin}}(j)$’s and $r_{k,\text{fin}}(j)$’s of the E-EnMLM/TH, via (25), (27), respectively.

5) Determine TC and CC probabilities via (20). Note that the calculation of CC probabilities of service-class $k$ calls, requires $N_k - 1$ sources.

6) Determine $U$ (in b.u.) via (21).

### 4.3. Extension of the E-EnMLM/TH to include adaptive quasi-random traffic (EA-EnMLM/TH)

To include adaptive traffic in the E-EnMLM/TH, let $K = |K_e| + |K_a|$, where $K_e$ and $K_a$ are the number of elastic and adaptive service-classes, respectively.

Consider again the tutorial example of Subsection 4.1 but now 2nd service-class calls are adaptive. Compared to Fig. 4, we see that in the new state transition diagram (that includes $r(n)$’s) of Fig. 6, the values of $\mu_2$ do not alter. As far as the modified state transition diagram (that includes $\phi_k(n)$’s) is concerned, it is similar to Fig. 5.

Following the analysis of Subsection 2.3, the values of $x(n)$’s and $\phi_k(n)$’s are given by (13) and (5), respectively while $G_{\text{fin}}(j)$’s can be determined by:

$$G_{\text{fin}}(j) = \begin{cases} 1, & \text{for } j = 0 \\ \frac{1}{\min(C, j)} \sum_{k \in K_e} (N_k - n_k^* + 1)a_{k,\text{fin}}b_k [G_{\text{fin}}(j - b_k) - T_{k,\text{fin}}(j - b_k)] \\ \frac{1}{\min(C, j)} \sum_{k \in K_a} (N_k - n_k^* + 1)a_{k,\text{fin}}b_k [G_{\text{fin}}(j - b_k) - T_{k,\text{fin}}(j - b_k)], & \text{for } j = 1, \ldots, T \\ 0, & \text{otherwise} \end{cases}$$

(29)

To calculate $T_{k,\text{fin}}(j)$ in (29) we consider two subsets: 1) $n_k^* b_k \leq j \leq C$ and 2) $C + 1 \leq j \leq T - b_k$.

For subset (1), we consider a system, of capacity $F_k = C - n_k^* b_k$, that accommodates all service-classes but service-class $k$ and define $r_{k,\text{fin}}(j)$, $j = 0, \ldots, F_k$. 

\[18\]
as follows:

\[ r_{k,\text{fin}}(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{2} \sum_{i \in K, k} (N_k - n_i + 1) a_{i, \text{fin}} b_i \left[ r_{k,\text{fin}}(j - b_i) - T_{i,\text{fin}}(j - b_i) \right], \\
\text{for } j = 1, \ldots, F_k \\
0, & \text{otherwise}
\] \quad (30)

Having determined \( r_{k,\text{fin}}(j) \), we compute the un-normalized values of \( T_{k,\text{fin}}(j) \)'s via (23) when \( n_k^* b_k \leq j \leq C \) and via (24) when \( C + 1 \leq j \leq T - b_k \).

To calculate \( G_{\text{fin}}(j) \)'s and \( r_{k,\text{fin}}(j) \)'s via (29) and (30), respectively, the knowledge of \( n_k \) in each state \( j \), \( n_k(j) \), is required. We approximate \( n_k(j) \) by the mean number of service-class \( k \) calls in state \( j \), \( y_k(j) \), when calls follow a Poisson process, i.e., \( n_k(j) \approx y_k(j) \). This means that the values of \( y_k(j) \) will be determined via the EA-EMLM/TH. Thus, to determine \( G_{\text{fin}}(j) \)'s and \( r_{k,\text{fin}}(j) \)'s we propose the formulas (31) and (34), respectively:

\[ G_{\text{fin}}(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{\min(C,j)} \sum_{k \in K_a} (N_k - y_k(j - b_k)) a_{k,\text{fin}} b_k \left[ G_{\text{fin}}(j - b_k) - T_{k,\text{fin}}(j - b_k) \right], \\
\text{for } j = 1, \ldots, T \\
0, & \text{otherwise}
\] \quad (31)

where the values of \( y_k(j) \)'s for elastic traffic are given by (32) while those for adaptive traffic are given by (33):

\[ y_k(j)G(j) = \begin{cases} 
\frac{1}{\min(C,j)} a_k b_k \left[ G(j - b_k) - T_k(j - b_k) \right] (y_k(j - b_k) + 1) + \\
\frac{1}{\min(C,j)} \sum_{i \in K_a, i \neq k} a_i b_i \left[ G(j - b_i) - T_i(j - b_i) \right] y_k(j - b_i) \\
\frac{1}{2} \sum_{i \in K_a} a_i b_i \left[ G(j - b_i) - T_i(j - b_i) \right] y_k(j - b_i)
\end{cases} \quad (32)

\[ y_k(j)G(j) = \begin{cases} 
\frac{1}{2} a_k b_k \left[ G(j - b_k) - T_k(j - b_k) \right] (y_k(j - b_k) + 1) + \\
\frac{1}{2} \sum_{i \in K_a, i \neq k} a_i b_i \left[ G(j - b_i) - T_i(j - b_i) \right] y_k(j - b_i) \\
+ \frac{1}{\min(C,j)} \sum_{i \in K_a} a_i b_i \left[ G(j - b_i) - T_i(j - b_i) \right] y_k(j - b_i)
\end{cases} \quad (33)
\[ r_{k,\text{fin}}(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{j} \sum_{i \in K \setminus k} (N_i - y_{i,k}(j-b_i))a_{i,\text{fin}}b_i [r_{k,\text{fin}}(j-b_i) - T_{i,\text{fin}}(j-b_i)], & \text{for } j = 1, \ldots, F_k \\
0, & \text{otherwise} 
\end{cases} \]  
(34)

where \( y_{i,k}(j) \) is determined by (28).

The proof of (32) and (33) is similar to that of Theorem 2 in [39] and thus is omitted. As far as the TC probabilities and the link utilization are concerned, they can be determined by (20) and (21), respectively.

The following algorithm summarizes the procedure for the calculation of TC probabilities, CC probabilities and link utilization in the EA-EnMLM/TH:

1) Determine \( G(j) \)’s, \( r_k(j) \)’s and \( T_k(j) \)’s of the corresponding Poisson model (EA-EMLM/TH) via (14), (10) and (11)-(12), respectively.

2) Determine \( y_k(j) \) of the EA-EMLM/TH via (32) and (33) for elastic and adaptive service-classes, respectively.

3) Determine \( T_{k,\text{fin}}(j) \)’s of the EA-EnMLM/TH, via (23) when \( n_k^* b_k \leq j \leq C \) and via (24) when \( C + 1 \leq j \leq T - b_k \).

4) Determine \( G_{\text{fin}}(j) \)’s and \( r_{k,\text{fin}}(j) \)’s of the EA-EnMLM/TH, via (31), (32), respectively.

5) Determine TC and CC probabilities via (20). Note that the calculation of CC probabilities of service-class \( k \) calls, requires \( N_k - 1 \) sources.

6) Determine \( U \) (in b.u.) via (21).

5. The proposed EA-EnMLM/TH including the BR policy

(EA-EnMLM/TH-BR)

Consider again a single link of capacity \( C \) b.u. that accommodates \( K(K = |K_e| + |K_a|) \) service-classes of quasi-random arriving calls. A new service-class \( k \) (\( k = 1, \ldots, K \)) call has a peak-bandwidth requirement of \( b_k \) b.u. and a BR parameter \( t_k \). If \( j + b_k \leq T - t_k \) and \( n_k + 1 \leq n_k^* \), then the call is accepted in the link. Otherwise the call is blocked and lost.
To determine \( G(j) \)'s in the EA-EnMLM/TH-BR we propose the formula:

\[
G_{\text{fin}}(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{\min(C,j)} \sum_{k \in K_a} (N_k - n_k + 1) a_{k,\text{fin}} D_k(j-b_k) [G_{\text{fin}}(j-b_k) - T_{k,\text{fin}}(j-b_k)] \\
+ \frac{1}{T} \sum_{k \in K_a} (N_k - n_k + 1) a_{k,\text{fin}} D_k(j-b_k) [G_{\text{fin}}(j-b_k) - T_{k,\text{fin}}(j-b_k)], & \text{for } j = 1, \ldots, T \\
0, & \text{otherwise} 
\end{cases}
\] (35)

where the values of \( D_k(j-b_k) \) are given by (16). If the link accommodates only elastic service-classes (E-EnMLM/TH-BR) then only the first summation of (35) should be used.

Similar to the EA-EMLM/TH-BR (Section 3), we determine the TC probabilities of service-class \( k \), \( P_{b_k} \), based on two groups of states as in (18):

\[
P_{b_k} = \sum_{j=T-b_k-t_k+1}^{T} G^{-1} G_{\text{fin}}(j) + \sum_{j \neq n^*_k b_k} G^{-1} T_{k,\text{fin}}(j) 
\] (36)

where \( G = \sum_{j=0}^{G_{\text{fin}}(j)} \) is the normalization constant.

The CC probabilities of service-class \( k \) calls can be given by (36) assuming \( N_k - 1 \) sources while the link utilization can be determined by (21).

In (35), (36) the knowledge of \( T_{k,\text{fin}}(j) \) is required for \( n^*_k b_k \leq j \leq T - b_k - t_k \). We consider again two subsets: 1) \( n^*_k b_k \leq j \leq C \) and 2) \( C + 1 \leq j \leq T - b_k - t_k \).

For subset (1), we can use (30) and (23) where \( F_k = T - b_k - t_k - n^*_k b_k \), while for subset (2) we can use (24) where the values of \( x(n) \) are given by (13) and

\[ \Omega_j = \{ n \notin \Omega : n^*_k b_k + \sum_{i=1, i \neq k}^{K} n_i b_i = j, \ C + 1 \leq j \leq T - b_k - t_k \}. \]

To calculate \( G_{\text{fin}}(j) \)'s via (35) we approximate \( n_k(j) \) by the mean number of service-class \( k \) calls in state \( j \), \( y_k(j) \), when calls follow a Poisson process, i.e.,

\( n_k(j) \approx y_k(j) \). This means that the values of \( y_k(j) \) will be determined via the
EA-EMLM/TH-BR. Thus, to determine $G_{\text{fin}}(j)$’s we propose the formula:

$$
G_{\text{fin}}(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{\min(C,j)} \sum_{k=1}^{K} [N_k - y_k(j-b_k)] a_{k,\text{fin}} D_k(j-b_k) [G_{\text{fin}}(j-b_k) - T_{k,\text{fin}}(j-b_k)] \\
+ \sum_{k=1}^{K} [N_k - y_k(j-b_k)] a_{k,\text{fin}} D_k(j-b_k) [G_{\text{fin}}(j-b_k) - T_{k,\text{fin}}(j-b_k)], & \text{for } j = 1, ..., T \\
0, & \text{otherwise}
\end{cases}
$$

(37)

where the values of $D_k(j-b_k)$ are given by (16) while the values of $y_k(j)$’s for elastic and adaptive traffic are given by (38), (39), respectively:

$$
y_k(j) G(j) = \frac{1}{\min(C,j)} a_k D_k(j-b_k) [G(j-b_k) - T_k(j-b_k)] (y_k(j-b_k)+1) \\
+ \frac{1}{\min(C,j)} \sum_{i=1, i \neq k}^{K} a_i D_i(j-b_i) [G(j-b_i) - T_i(j-b_i)] y_k(j-b_i) + \frac{1}{\min(C,j)} \sum_{i=1, i \neq k}^{K} a_i D_i(j-b_i) [G(j-b_i) - T_i(j-b_i)] y_k(j-b_i)
$$

(38)

$$
y_k(j) G(j) = \frac{1}{\min(C,j)} a_k D_k(j-b_k) [G(j-b_k) - T_k(j-b_k)] (y_k(j-b_k)+1) \\
+ \frac{1}{\min(C,j)} \sum_{i=1, i \neq k}^{K} a_i D_i(j-b_i) [G(j-b_i) - T_i(j-b_i)] y_k(j-b_i) \\
+ \frac{1}{\min(C,j)} \sum_{i=1}^{K} a_i D_i(j-b_i) [G(j-b_i) - T_i(j-b_i)] y_k(j-b_i)
$$

(39)

The proof of (38) and (39) is similar to that of Theorem 2 in [36] and thus is omitted.

6. Numerical results - Evaluation

In this section, we present numerical results for the proposed models (E-EMLM/TH-BR, EA-EMLM/TH-BR, E-EnMLM/TH-BR and EA-EnMLM/TH-BR) and the models of [39], [40], [46] (EA-EMLM, EA-EnMLM, EMLM/TH, respectively) and [47] (EnMLM/TH, EMLM/TH-BR, EnMLM/TH-BR). Through the proposed models we obtain analytical TC probabilities and link utilization results, and compare them with the corresponding simulation results, in order to reveal the accuracy of the proposed models. The analytical and simulation
results of CC probabilities are of similar accuracy to those given for TC probabilities and therefore are not presented. The simulation model is based on the bandwidth compression/expansion mechanism described by $r(n)$'s. On the other hand, the proposed analytical models are based on $\phi_k(n)$'s. Simulation results are mean values of 7 runs. Each run is based on the generation of four million calls. To account for a warm-up period, the blocking events of the first 5% of these generated calls are not considered in the results. Due to the fact that the confidence intervals are very small, they are not presented in the figures that follow. The simulation language used is Simscript III [72].

As an application example, we consider a link of capacity $C = 70$ b.u. and three values of $T$: 1) $T = C = 70$ b.u., 2) $T = 75$ b.u. with $r_{\text{min}} = C/T = 70/75$ and 3) $T = 80$ b.u. with $r_{\text{min}} = C/T = 70/80$. The link accommodates calls of three service-classes, with the following traffic characteristics:

1st class: $a_1 = 5.0$ erl, $N_1 = 30$ sources, $a_{1,\text{fin}} = a_1/N_1$ erl, $b_1 = 2$ b.u., $n_1^* = 25$

2nd class: $a_2 = 1.5$ erl, $N_2 = 30$ sources, $a_{2,\text{fin}} = a_2/N_2$ erl, $b_2 = 5$ b.u., $n_2^* = 11$

3rd class: $a_3 = 1.0$ erl, $N_3 = 30$ sources, $a_{3,\text{fin}} = a_3/N_3$ erl, $b_3 = 9$ b.u., $n_3^* = 6$

The BR parameters for the three service-classes are the following: $t_1 = 7$, $t_2 = 4$ and $t_3 = 0$ b.u. Note that in the E-EnMLM/TH and the E-EnMLM/TH-BR all service-classes are elastic while in the EA-EnMLM/TH and the EA-EnMLM/TH-BR the first service-class is elastic and the other two are adaptive.

In the x-axis of all figures, traffic loads $a_1, a_2$ and $a_3$ increase in steps of 1, 0.5 and 0.25 erl, respectively. So, Point 1 represents the vector $(a_1, a_2, a_3) = (5.0, 1.5, 1.0)$, while Point 7 is $(a_1, a_2, a_3) = (11.0, 4.5, 2.5)$.

In Figs. 7-9, we consider the proposed E-EnMLM/TH and present the analytical and simulation TC probabilities results of the three service-classes, respectively, for all values of $T$. For comparison, we present the corresponding analytical results of the EMLM/TH and the EnMLM/TH (when $T = C = 70$). According to Figs. 7-9, we deduce that:

(i) the results obtained by the proposed formulas are very close to the simulation results.
(ii) The bandwidth compression mechanism reduces TC probabilities of all service-classes as expected (higher reduction is achieved for $T=80$ b.u.). This fact shows the consistency of the proposed model.

(iii) The analytical TC probabilities results obtained by the EMLM/TH and EnMLM/TH fail to approximate the simulation TC probabilities results of the E-EnMLM/TH. This fact reveals the necessity of the proposed model.

In Figs. 10-11, we consider the proposed E-EMLM/TH-BR, E-EnMLM/TH-BR and present the analytical and simulation TC probabilities results of the 1st and 3rd service-classes, respectively, for all values of $T$. The results obtained for the 2nd service-class are of similar accuracy and therefore are omitted. For comparison, we present the corresponding analytical results of the EMLM/TH-BR and the EnMLM/TH-BR (when $T=C=70$). According to Figs. 10-11, our conclusions are similar to (i)-(iii). In addition, the application of the BR policy in the proposed models results in the CBP increase of the 1st and 2nd service-classes and the CBP decrease of the 3rd service-class. This behavior is expected since the BR parameters favor 3rd service-class calls. Similar conclusions between the analytical and simulation results are obtained in the EA-EMLM/TH-BR and EA-EnMLM/TH-BR, and thus these results are omitted.

In Figs. 12-13, we present the link utilization results (in b.u.), with (Fig. 13) or without (Fig. 12) the BR policy. Again, the analytical results are very close to simulation, while the existing EMLM/TH, EnMLM/TH, EMLM/TH-BR and the EnMLM/TH-BR fail to approximate the results obtained by the corresponding proposed models.

In Figs. 14-15, we consider the E-EnMLM/TH and EA-EnMLM/TH together with the existing EnMLM/TH and present the analytical TC probabilities results of the 1st and 3rd service-classes, respectively, for $T = 75$ b.u. The simulation results of the EA-EnMLM/TH are very close to the corresponding analytical results and therefore are not presented. To show the necessity of the proposed models, we consider two values of $n_3^* = 4, 5$ calls. The existing EnMLM/TH fails to approximate the results obtained by the proposed models, when $n_3^* = 4, 5$. The fact that the EA-EnMLM/TH gives slightly better
results compared to the E-EnMLM/TH is explained by the fact that in the EA-EnMLM/TH calls of the 2nd and the 3rd service-class are adaptive and thus spend less service time in the system compared to the case of the E-EnMLM/TH. In addition, the increase of \( n_3^* \) from 4 to 5 calls, results in the increase of TC probabilities for the 1st (Fig. 14) and 2nd service-classes and the decrease of TC probabilities for the 3rd service-class (Fig. 15). All these results show the consistency of the proposed models.

Similar behavior appears in Figs. 16-17, whereby we consider the proposed EA-EnMLM/TH together with the existing EA-EMLM and EA-EnMLM and present the analytical TC probabilities results of the 1st and 3rd service-classes, respectively, for \( T = 75 \) b.u. and \( T = 80 \) b.u. and \( n_3^* = 4, 5 \) calls. It is apparent that the EA-EMLM and EA-EnMLM fail to approximate the proposed models.

As a final comment, the results obtained by the proposed formulas are very close to the simulation results even for quite large values of \( T \) compared to \( C \) (e.g., \( T = 140 \) b.u.). However, increasing \( T \) results in a delay increase of elastic calls, which may be unacceptable for some applications. Thus, \( T \) should be chosen so that this delay remains within acceptable levels.

7. Conclusion

In this paper we propose multirate loss models where Poisson or quasi-random arriving calls of different service-classes compete for the available link bandwidth under the threshold and the bandwidth reservation policies. Calls can be elastic or adaptive. Both types can tolerate bandwidth compression, but only elastic calls should increase their service-time when their bandwidth is compressed. The analysis of the proposed models leads to approximate but recursive formulas for the calculation of the steady-state probabilities and consequently TC probabilities, CC probabilities and link utilization. Simulation results verify the accuracy of the proposed models. In addition, numerical results show the necessity of the proposed models, since existing models fail to approximate the results obtained by the proposed models, and their consistency.
Acknowledgment

The authors would like to thank the anonymous reviewers for their careful reading of our manuscript and their valuable comments and suggestions.

References


[8] I. Moscholios, P. Nikolaropoulos and M. Logothetis, “Call level blocking of ON-OFF traffic sources with retrials under the complete sharing policy”, *Proc. 18th ITC*, Berlin, Germany, Sept. 2003, pp. 811-820.


Table 1: State space and occupied link bandwidth.

<table>
<thead>
<tr>
<th>n1</th>
<th>n2</th>
<th>j (before bandwidth compression)</th>
<th>j (after bandwidth compression)</th>
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</thead>
<tbody>
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<td></td>
<td></td>
<td>0 ≤ j ≤ T</td>
<td>0 ≤ j ≤ C</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
</tr>
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<td>4</td>
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</tr>
<tr>
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<td>4</td>
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Table 2: Values of state dependent factors (elastic traffic).

<table>
<thead>
<tr>
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<th>n2</th>
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<th>(\phi_2(n))</th>
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</tr>
<tr>
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<td>0</td>
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<td>0.667</td>
<td>1.000</td>
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</table>
Table 3: Values of state dependent factors (elastic and adaptive traffic).

<table>
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<th>$n_1$</th>
<th>$n_2$</th>
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<th>$\phi_1(n)$</th>
<th>$\phi_2(n)$</th>
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</thead>
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Figure 1: State transition diagram of the tutorial example for elastic random traffic with $r(n)$’s.

Figure 2: Modified state transition diagram of the tutorial example for elastic random traffic with $\phi_k(n)$’s.
Figure 3: State transition diagram of the tutorial example for elastic and adaptive random traffic with $r(n)$'s.

Figure 4: State transition diagram of the tutorial example for elastic quasi-random traffic with $r(n)$'s.
Figure 5: Modified state transition diagram of the tutorial example for elastic quasi-random traffic with $\phi_k(n)$'s.

Figure 6: State transition diagram of the tutorial example for elastic and adaptive quasi-random traffic with $r(n)$'s.
Figure 7: TC probabilities of the 1st service-class, when $n_1^* = 6$ (under the TH policy only).
Figure 8: TC probabilities of the 2nd service-class, when $n^*_3 = 6$ (under the TH policy only).
Figure 9: TC probabilities of the 3rd service-class, when $n^*_3 = 6$ (under the TH policy only).

Figure 9: TC probabilities of the 3rd service-class, when $n^*_3 = 6$ (under the TH policy only).
Figure 10: TC probabilities of the 1st service-class, when $n_3^* = 6$ (under both policies).
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Figure 12: Link Utilization, when $n^*_3 = 6$ (under the TH policy only).
Figure 13: Link Utilization, when $n^*_3 = 6$ (under both policies).
Figure 14: TC probabilities of the 1st service-class, when $n_3^* = 4, 5$ and $T = 75$ (EnMLM/TH vs E-EnMLM/TH vs EA-EnMLM/TH).
Figure 15: TC probabilities of the 3rd service-class, when $n^*_3 = 4, 5$ and $T = 75$ (EnMLM/TH vs E-EnMLM/TH vs EA-EnMLM/TH).
Figure 16: TC probabilities of the 1st service-class, when $n_3^* = 4, 5$ and $T = 75, 80$ (EA-EMLM vs EA-EnMLM vs EA-EnMLM/TH).
Figure 17: TC probabilities of the 3rd service-class, when $n^*_3 = 4, 5$ and $T = 75, 80$ (EA-EMLM vs EA-EnMLM vs EA-EnMLM/TH).
Figure 18: State transition diagram of the E-EMLM/TH.

\[ \sum_{k=1}^{K} \lambda_k \delta_k^-(n)P(n^-_k) \]
\[ \sum_{k=1}^{K} \lambda_k \delta_k^+(n)P(n^+_k) \]
\[ \sum_{k=1}^{K} n_k \mu_k \delta_k^-(n)\phi_k(n)P(n) \]
\[ \sum_{k=1}^{K} (n_k + 1) \mu_k \delta_k^+(n)\phi_k(n^+_k)P(n^+_k) \]

Figure 19: State transition diagram of the E-EnMLM/TH.

\[ \sum_{k=1}^{K} (N_k - n_k + 1) \nu_k \delta_k^-(n)P_{fin}(n^-) \]
\[ \sum_{k=1}^{K} (N_k - n_k) \nu_k \delta_k^+(n)P_{fin}(n^+) \]
\[ \sum_{k=1}^{K} n_k \mu_k \delta_k^-(n)\phi_k(n)P_{fin}(n) \]
\[ \sum_{k=1}^{K} (n_k + 1) \mu_k \delta_k^+(n)\phi_k(n^+_k)P_{fin}(n^+_k) \]
Appendix A. Proof of Theorem 1

The steady state transition rates of the E-EMLM/TH are shown in Fig. 18, whereby the global balance equation for state \( n = (n_1, ..., n_{k-1}, n_k, n_{k+1}, ..., n_K) \), expressed as \( \text{rate into state } n = \text{rate out of state } n \), is:

\[
\sum_{k=1}^{K} \lambda_k \delta^+(n)P(n^-) + \sum_{k=1}^{K} (n_k + 1)\mu_k \delta^+(n)\phi_k(n^+)P(n^+) = \sum_{k=1}^{K} \lambda_k \delta^-(n)P(n) + \sum_{k=1}^{K} n_k \mu_k \delta^-\phi_k(n)P(n^-) \tag{A.1}
\]

where:

\[
n^+_k = (n_1, ..., n_{k-1}, n_k + 1, n_{k+1}, ..., n_K), \quad n^-_k = (n_1, ..., n_{k-1}, n_k - 1, n_{k+1}, ..., n_K)
\]

\[
\delta^+(n) = \begin{cases} 
1 & \text{if } n^+_k \in \Omega \\
0 & \text{otherwise}
\end{cases}, \quad \delta^-(n) = \begin{cases} 
1 & \text{if } n^-_k \in \Omega \\
0 & \text{otherwise}
\end{cases}, \quad \text{and } P(n), P(n^-_k), P(n^+_k)
\]

are the probability distributions of states \( n, n^-_k, n^+_k \), respectively.

Assume now, the existence of Local Balance (LB) between adjacent states. Equations (A.2) and (A.3) are the detailed LB equations which exist because the Markov chain of the modified model is reversible. Then, based on Fig. 18, the following LB equations (rate up = rate down) are extracted:

\[
\lambda_k \delta^-\phi_k(n)P(n^-) = n_k \mu_k \delta^+(n)\phi_k(n)P(n) \tag{A.2}
\]

\[
\lambda_k \delta^+(n)P(n) = (n_k + 1)\mu_k \delta^-\phi_k(n^+)P(n^+) \tag{A.3}
\]

for \( k = 1, ..., K \) and \( n \in \Omega \).

Based on the LB assumption, the probability distribution \( P(n) \) has the solution:

\[
P(n) = G^{-1}\left(x(n) \prod_{k=1}^{K} \frac{a_k^{n_k}}{n_k!}\right) \tag{A.4}
\]

where \( a_k = \frac{\lambda_k}{\mu_k} \) is the offered traffic-load (in erl) of service-class \( k \), \( G \) is the normalization constant given by \( G \equiv G(\Omega) = \sum_{n \in \Omega} x(n) \prod_{k=1}^{K} \frac{a_k^{n_k}}{n_k!} \) and \( \Omega = \{ n: 0 \leq nb \leq T, n_k \leq n^*_k, k = 1, ..., K \} \). Note that although the Markov chain
becomes reversible, the probability distribution \( P(n) \) does not have a PFS due to the summation of (6) needed for \( x(n) \).

We now define \( G(j) \), as:

\[
G(j) = \sum_{n \in \Omega_j} P(n) \tag{A.5}
\]

where \( \Omega_j \) is the set of states whereby exactly \( j \) b.u. are occupied, i.e., \( \Omega_j = \{n \in \Omega : nb = j\} \).

Consider now two sets: 1) \( 0 \leq j \leq C \) and 2) \( C < j \leq T \). For set (1), we have the EMLM/TH [43]. In that case, no bandwidth compression takes place and \( G(j) \) is determined by the following exact recursive formula [46]:

\[
G(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{j} \sum_{k=1}^{j} a_k b_k [G(j - b_k) - T_k(j - b_k)], & \text{for } j = 1, \ldots, C \\
0, & \text{otherwise} 
\end{cases} \tag{A.6}
\]

where \( T_k(x) \) is the un-normalized probability that \( x \) b.u. are occupied while the number of service-class \( k \) calls is \( n_k^* \), or:

\[
T_k(x) := Pr(j = x, n_k = n_k^*) \tag{A.7}
\]

In (A.7) the fact that \( n_k = n_k^* \) implies that:

1. \( j \geq n_k^* b_k \) and therefore \( T_k(x) = 0 \) for \( x = 0, 1, \ldots, n_k^* b_k - 1 \), and
2. \( T_k(x) \) is a blocking probability factor for service-class \( k \) calls.

For set (2), i.e., when \( C < j \leq T \), we substitute (5) in (A.2) to have:

\[
a_k x(n) P(n_k^-) = n_k x(n_k^-) P(n) \tag{A.8}
\]

where \( \delta_k(n) = 1 \), since the TH policy is a coordinate convex policy [37].

**Note:** The basic characteristic of a coordinate convex policy is that if \( n \in \Omega \) and \( n_k \geq 1 \) then the state \( n_k^- = (n_1, \ldots, n_{k-1}, n_k - 1, n_{k+1}, \ldots, n_K) \) is also an admissible state.

Multiplying both sides of (A.8) by \( b_k \) and summing over \( k \), we obtain:

\[
x(n) \sum_{k=1}^{K} a_k b_k P(n_k^-) = P(n) \sum_{k=1}^{K} n_k b_k x(n_k^-) \tag{A.9}
\]
Equation (A.9), due to (6) is written as:

\[ P(n) = \frac{1}{C} \sum_{k=1}^{K} a_k b_k P(n_k) \]  

(A.10)

Summing both sides of (A.10) over \( \Omega_j = \{ n \in \Omega : nb = j \} \) and based on (A.5), we obtain:

\[ G(j)C = \sum_{k=1}^{K} a_k b_k \sum_{n \in \Omega_j} P(n_k) \]  

(A.11)

Since the number of in-service calls of service-class \( k \) should not exceed \( n_k^* \) the term \( \sum_{n \in \Omega_j} P(n_k) = G(j - b_k) - Pr[x = j - b_k, n_k = n_k^*] \). Thus, (A.11) can be written as:

\[ G(j) = \frac{1}{C} \sum_{k=1}^{K} a_k b_k [G(j - b_k) - T_k(j - b_k)] \]  

(A.12)

where \( T_k(x) \) is given by (A.7).

The combination of (A.6) and (A.12) results in (7). End of Proof.

Appendix B. Proof of Theorem 2

The graphical representation of the modified state transition diagram is similar to Fig. 2. The derivation of (13) is based on the following assumptions:

a) The bandwidth of all in-service calls of service-class \( k \), \( 1 \leq k \leq K \) (elastic or adaptive) is compressed by a factor \( \phi_k(n) \), in state \( C < nb \leq T \), so that:

\[ \sum_{k \in K_e} n_k b'_k + \sum_{k \in K_a} n_k b'_k = C \]  

(B.1)

b) The product service time by bandwidth per call of service-class \( k \) calls, \( k \in K \), remains the same in state \( n \) either of the irreversible or the reversible Markov chain. In other words:

For elastic service – classes:

\[ \frac{b_k r(n)}{\mu_k r(n)} = \frac{b'_k}{\mu_k \phi_k(n)} \Rightarrow \frac{b'_k}{b_k} = \phi_k(n) \]  

(B.2)

For adaptive service – classes:

\[ \frac{b_k r(n)}{\mu_k} = \frac{b'_k}{\mu_k \phi_k(n)} \Rightarrow \frac{b'_k}{b_k} = \phi_k(n) r(n) \]  

(B.3)

Under these assumptions, we can derive (13) by substituting (B.2), (B.3) and (5) into (B.1). End of Proof.
Appendix C. Proof of Theorem 3

The steady state transition rates of the E-EnMLM/TH are shown in Fig. 19, whereby the global balance equation for state \( n = (n_1, ..., n_{k-1}, n_k, n_{k+1}, ..., n_K) \), expressed as rate into state \( n = \text{rate out of state } n \), is:

\[
K \sum_{k=1}^{K} (N_k - n_k + 1)v_k \delta_k^+ (n)P_{fin}(n_k^-) + \sum_{k=1}^{K} (n_k + 1)\mu_k \delta_k^- (n) \phi_k(n_k^+)P_{fin}(n_k^+) - \\
K \sum_{k=1}^{K} (N_k - n_k)v_k \delta_k^- (n)P_{fin}(n) + \sum_{k=1}^{K} n_k \mu_k \delta_k^- (n) \phi_k(n)P_{fin}(n) = 0
\]

(C.1)

where:

\( n_k^+ = (n_1, ..., n_{k-1}, n_k + 1, n_{k+1}, ..., n_K) \), \( n_k^- = (n_1, ..., n_{k-1}, n_k - 1, n_{k+1}, ..., n_K) \)

while \( P_{fin}(n), P_{fin}(n_k^-), P_{fin}(n_k^+) \) are the probability distributions of states \( n, n_k^-, n_k^+ \), in the finite source model, respectively.

Based on Fig. 19, the following LB equations are extracted:

\[
(N_k - n_k + 1)v_k \delta_k^- (n)P_{fin}(n_k^-) = n_k \mu_k \delta_k^- (n) \phi_k(n)P_{fin}(n) \quad \text{(C.2)}
\]

\[
(N_k - n_k)v_k \delta_k^- (n)P_{fin}(n) = (n_k + 1)\mu_k \delta_k^+ (n) \phi_k(n_k^+)P_{fin}(n_k^+) \quad \text{(C.3)}
\]

for \( k = 1, ..., K \) and \( n \in \Omega \).

Based on the LB assumption, \( P_{fin}(n) \) has the solution:

\[
P_{fin}(n) = G^{-1} \left( x(n) \prod_{k=1}^{K} \left( \begin{array}{c} N_k \\ n_k \end{array} \right) a_{k,fin}^{n_k} \right) \quad \text{(C.4)}
\]

where \( a_{k,fin} = v_k/\mu_k \) is the offered traffic-load (in erl) per idle service-class \( k \) source, \( G \equiv G(\Omega) = \sum_{n \in \Omega} \left( x(n) \prod_{k=1}^{K} \left( \begin{array}{c} N_k \\ n_k \end{array} \right) a_{k,fin}^{n_k} \right) \), \( x(n) \) is given by (6) and \( \Omega = \{ n : 0 \leq nb \leq T, n_k \leq n_k^* \leq N_k, k = 1, ..., K \} \).

We now define \( G_{fin}(j) \) as:

\[
G_{fin}(j) = \sum_{n \in \Omega_j} P_{fin}(n)
\]

(C.5)

where \( \Omega_j = \{ n \in \Omega : nb = j \} \).
Consider now two sets: (1) \(0 \leq j \leq C\), and (2) \(C < j \leq T\). For set (1), we have the EnMLM/TH [47]. In that case, \(G_{\text{fin}}(j)\) is determined by an accurate and recursive formula [47]:

\[
G_{\text{fin}}(j) = \begin{cases} 
1, & \text{for } j = 0 \\
\frac{1}{C} \sum_{k=1}^{K} (N_k - n_k + 1)a_{k, \text{fin}}b_k[G_{\text{fin}}(j-b_k) - T_{k, \text{fin}}(j-b_k)], & \text{for } j = 1, \ldots, C \\
0, & \text{otherwise}
\end{cases}
\]  

(C.6)

where \(T_{k, \text{fin}}(x)\) is given by:

\[
T_{k, \text{fin}}(x) := \Pr(j = x, n_k = n_k^*)
\]  

(C.7)

For set (2), i.e., when \(C < j \leq T\), we substitute (5) in (C.2) to have:

\[
(N_k - n_k + 1)a_{k, \text{fin}}b_kP_{\text{fin}}(n_k^*) = n_kx(n_k^*)P_{\text{fin}}(n_k^*)
\]  

(C.8)

where \(\delta_k^-(n) = 1\) since the TH policy is a coordinate convex policy.

Multiplying both sides of (C.8) by \(b_k\) and summing over \(k\), we obtain:

\[
x(n_k) \sum_{k=1}^{K} (N_k - n_k + 1)a_{k, \text{fin}}b_kP_{\text{fin}}(n_k^*) = \sum_{k=1}^{K} n_kb_kx(n_k^*)
\]  

(C.9)

Equation (C.9), due to (6) is written as:

\[
P_{\text{fin}}(n_k) = \frac{1}{C} \sum_{k=1}^{K} (N_k - n_k + 1)a_{k, \text{fin}}b_kP_{\text{fin}}(n_k^*)
\]  

(C.10)

Summing both sides of (C.10) over \(\mathcal{N}_j = \{ n \in \Omega : nb = j \} \), and based on (C.5), we get:

\[
G_{\text{fin}}(j)C = \sum_{n \in \mathcal{N}_j} (N_k - n_k + 1)P_{\text{fin}}(n_k^*)
\]  

(C.11)

The term \(\sum_{n \in \mathcal{N}_j} (N_k - n_k + 1)P_{\text{fin}}(n_k^*)\) in (C.11) is written as:

\[
\sum_{n \in \mathcal{N}_j} (N_k - n_k + 1)P_{\text{fin}}(n_k^*) = N_k \sum_{n \in \mathcal{N}_j} P_{\text{fin}}(n_k^-) - \sum_{n \in \mathcal{N}_j} (n_k - 1)P_{\text{fin}}(n_k^-)
\]  

(C.12)

Since \(n_k \leq n_k^*\), the term \(\sum_{n \in \mathcal{N}_j} P_{\text{fin}}(n_k^-) = G_{\text{fin}}(j-b_k) - \Pr[x = j - b_k, n_k = n_k^*]\).

Thus, the first term of the right hand side of (C.12) becomes:

\[
N_k \sum_{n \in \mathcal{N}_j} P_{\text{fin}}(n_k^-) = N_k [G_{\text{fin}}(j-b_k) - \Pr[x = j - b_k, n_k = n_k^*]]
\]  

(C.13)
The second term of the right hand side of (C.13) becomes:

$$\sum_{n \in \Omega} (n_k - 1) P_{\text{fin}}(n_k) = y_{k, \text{fin}}(j - b_k) \left[ G_{\text{fin}}(j - b_k) - \Pr [x = j - b_k, n_k = n_k^*] \right] \quad (C.14)$$

where $y_{k, \text{fin}}(j - b_k)$ is the average number of service-class $k$ calls in state $j - b_k$.

Based on (C.13) and (C.14), (C.12) becomes:

$$\sum_{n \in \Omega} (N_k - n_k + 1) P_{\text{fin}}(n_k) = (N_k - y_{k, \text{fin}}(j - b_k)) \left[ G_{\text{fin}}(j - b_k) - T_{k, \text{fin}}(j - b_k) \right] \quad (C.15)$$

where: $T_{k, \text{fin}}(j - b_k) = \Pr (x = j - b_k, n_k = n_k^*)$.

Equation (C.11) due to (C.15) is written as:

$$G_{\text{fin}}(j) = \frac{1}{C} \sum_{k=1}^{K} a_{k, \text{fin}} b_k (N_k - y_{k, \text{fin}}(j - b_k)) \left[ G_{\text{fin}}(j - b_k) - T_{k, \text{fin}}(j - b_k) \right] \quad (C.16)$$

In order to determine $y_{k, \text{fin}}(j - b_k)$'s in (C.16) we use a transformation already proposed in a lemma of [4] for the case of the EnMLM. According to [4], two stochastic systems with: a) the same traffic description parameters $(K, N_k, a_{k, \text{fin}})$, $k = 1, ..., K$ and b) exactly the same set of states are equivalent in the sense that they result in the same CBP. The bandwidth requirements and the capacity $C$ of the equivalent stochastic system can be chosen in such a way so that conditions (a) and (b) are valid. No other restriction is necessary. This means, that we have the freedom to choose $b_k$'s and $C$ in a way that they result in an equivalent stochastic system whereby each state in $\Omega$ has a unique occupancy $j$. In that case, each state $j$ of the equivalent stochastic system is reached only by the previous state $j - b_k$ and therefore we have $y_{k, \text{fin}}(j - b_k) = n_k - 1$ and $y_{k, \text{fin}}(j) = n_k$. The interested reader may resort to subsection 2.1.1 of [70] for a detailed example of such a transformation in the case of the EnMLM. Based on the above, (C.16) becomes:

$$G_{\text{fin}}(j) = \frac{1}{C} \sum_{k=1}^{K} (N_k - n_k + 1) a_{k, \text{fin}} b_k \left[ G_{\text{fin}}(j - b_k) - T_{k, \text{fin}}(j - b_k) \right] \quad (C.17)$$

The combination of (C.6) and (C.17) results in (19). **End of Proof.**
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