

The Generalized Max-Min Fairness Policy for Elastic Traffic based on Linear Programming

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Abstract Most of elastic traffic calls require a rate between certain limits, while it is essential for a bandwidth allocation mechanism to allocate the available bandwidth in a fair manner among the elastic calls. Therefore, the min and max bandwidth requirements of calls have to be taken into account in the bandwidth allocation process. Such a process is the Generalized Max-Min Fairness policy (GMM). It has been described without mathematical support in a five-step procedure, in which the fairness criteria are given without coming into details. In this paper, we propose a new algorithm for the GMM policy, in a clear mathematical way, based on Linear Programming (LP), and it is directly convertible into computer software. To clarify the proposed new algorithm, we present selected numerical examples on a well-established testbed network.

1. Introduction

Elastic traffic sources, such as Internet traffic sources using TCP and sources using the Available Bit Rate (ABR) service in ATM networks, can adjust their rates according to a flow/congestion control mechanism, which allocates to them the available bandwidth left over from non-elastic services. This bandwidth needs to be allocated fairly among competing elastic traffic sources (calls), thus guaranteeing that the quality of service in the network remains in an acceptable level [1].

The most popular Fair Bandwidth Allocation (FBA) policy is the Max-Min Fairness (MMF) policy [2], initially adapted by the ATM Forum to allocate available network bandwidth among ABR connections. In the MMF policy, a bandwidth allocation is said to be max-min fair, if the bandwidth allocated to a connection cannot be increased without decreasing at the same time the rate of a connection having a smaller or equal bandwidth allocated (MMF criterion). The main “drawback” of the MMF policy is that it does not take into account the Minimum Cell Rate (MCR) and Peak Cell Rate (PCR) elastic traffic description parameters. This drawback has been well faced by an extension of the MMF policy, the Generalized Max-Min (GMM) [3], which supports MCR and PCR for each call connection.

The GMM policy has been described in procedural steps, by expressing the philosophy of the

GMM policy, without coming into details. This has the advantage of an easy presentation, but fails to describe the policy in a clear mathematical way. The latter can be achieved with the aid of the Linear Programming (LP), which shows clearly the optimization function and the set of constraints. A simple LP model has already been proposed ([4],[5]) for the description of the MMF policy (without the MCR, PCR or weights). Unfortunately, the GMM policy cannot be applied through the MMF LP model, simply by adding the traffic parameters (MCR, PCR) constraint-set. For the description of the GMM policy through LP, we propose a new algorithm. First, we present it in procedural steps (without coming into details), by expressing the key ideas (philosophy of our algorithm) of how the LP model is formulated. This is in order to show the basic differences between the new and the initial GMM algorithm. On the other hand this comparison shows that the new algorithm describes in fact the GMM policy. Then, we give the detailed algorithm in the form of flow-chart. Although the new algorithm is more complicated than the initial GMM procedure ([3]), it has the advantage of the direct computer implementation.

The structure of this paper is as follows. In section 2, we review the GMM policy and include the initial step-by-step procedure whereby this policy can be described. In section 3 we propose the description of the GMM policy through an LP model. We present a step-by-step procedure describing our methodology in a similar way to the initial step-by-step procedure. We discuss the main differences between them and afterwards we give the exact algorithm (in the form of flow-chart) for the LP model formulation. Section 4 is the numerical section, where we give application examples considering a well-known testbed network. We conclude in section 5.

2. Overview of the GMM policy

The GMM policy satisfies the MCR of each connection and then tries to maximize the rate of the smallest connection among all connections, while satisfying this connection’s PCR. Given the best smallest (max) rate allocation, it continues by maximizing the rate of the connection with the second smallest rate and so forth. The GMM rate allocation algorithm can be described in 5 steps [3]:

1. Start the rate of each connection with its MCR.
2. Sort all connections in an increasing rate order.
3. Increase the rate of the connection with the smallest rate among all connections until one of the following events takes place:
 - The rate of this connection reaches the second smallest rate among all connections;
 - A link saturates;
 - The connection's rate reaches its PCR.
4. If a link saturates or the connection's rate reaches its PCR in the previous step, remove from the network: a) the connections that either traverse the saturated link or reach their PCRs (respectively) and b) the network capacity associated with these (the removed) connections.
5. If there is no connection left, the algorithm terminates; otherwise it goes back to step 3 for the remaining connections and network capacity.

3. LP description of the GMM policy - Proposed new algorithm

In order to describe mathematically the GMM policy through an LP model, the following notation is necessary: Let L be the set of network links, S be the set of calls accommodated in the network and S_l be the set of calls which use link $l \in L$. Calls are conveyed by a fixed routing scheme. A call $s \in S$ may require a number of connections, n_s , of the same bandwidth requirements; that is, in this paper, a call is considered as a group of connections with the same bandwidth requirements. We denote by C_l the bandwidth-capacity of link $l \in L$, and by r_s , the bandwidth (rate) which is allocated to each connection of call $s \in S$ (the decision variable of the LP model). The final value of r_s (allocated rate) is denoted by r_s^* . The value r_s^* should be between the minimum, $r_{s,min}$, and the maximum, $r_{s,max}$, bandwidth requirements of call $s \in S$, whereas the vector $\mathbf{r}^* = (r_1^*, r_2^*, \dots, r_s^*)$ should satisfy the MMF criterion. Furthermore, the vectors $\mathbf{r}_{min} = (r_{1,min}, r_{2,min}, \dots, r_{s,min})$ and $\mathbf{r}_{max} = (r_{1,max}, r_{2,max}, \dots, r_{s,max})$ contain the minimum and the maximum permissible rates of all network calls, respectively.

The LP model formulation is distinguished in two phases, initial and final. In the initial phase, the LP model consists of two constraint sets, which are readily defined: the Link Capacity constraints (Set 1), and the Traffic Parameters constraints (Set 2). In the final phase, the minimum permissible rates in Set 2 are modified, while the LP model includes one more set of constraints, the Fairness constraints (Set 3). It is a set of equality constraints between the rates of the connections (in the initial phase of the LP model, Set 3 is empty) and is defined by the proposed new algorithm together with the minimum rates modifications in Set 2. The modifications in Set 2 are done so that the finally allocated rates satisfy the MMF criterion. The objective function of the LP model is to maximize the sum of connections rates.

The constraint sets:

$$\text{Link Capacity Set: } \sum_{s \in S_l} n_s r_s \leq C_l, \text{ for all } l \in L \quad (1)$$

$$\text{Traffic Parameters Set: } r_{s,min} \leq r_s \leq r_{s,max} \text{ for all } s \in S \quad (2)$$

$$\text{Fairness Set: } \text{null} \quad (3)$$

$$\text{Objective function: } \sum_{s \in S} r_s \Rightarrow \max \quad (4)$$

The proposed new GMM rate allocation algorithm is described in 8 steps:

0: Define the **Initial** phase of the (above) **LP model**.

1: Set $\mathbf{r} = \mathbf{r}_{min}$

2: In a vector R , sort the rates of all connections, in the order of increasing rate, including:

- **Minimum** rates
- **Maximum** rates
- **Quantized** values of the available **link bandwidth**, according to the number of connections that traverse the link (see Fig. 1)

3: **Increase** the rate of all connections to the next level (value) of R , while satisfying the minimum and maximum rates of each connection, until the rates reach the upper level of R , R_{max} , or some link saturates. R_{max} is not defined as the maximum value of R , but as: a) the maximum value of minimum rates when the current rates are not equal to one another, or b) the maximum value of the quantized values of the available bandwidth, when the current rates are equal to one another.

4: **If** some **link saturates then remove** from the network the connections that traverse that link as well as the network capacity occupied by such connections and return to step 1 via step 5.

5: **If** the connections of two different calls have the **same rate** (less than their maximum one) and traverse the same link **then add** an **equality constraint** between them in the Fairness set (Set 3 of the LP model) and return to step 1 after updating the minimum rates with the current rates ($\mathbf{r}_{min} = \mathbf{r}$).

6: Having reached the rates up to the level R_{max} , **solve** the initial LP model (i.e. find \mathbf{r}^*), as it has been modified up to this step (basically in Traffic Parameters set and Fairness set).

7: **Give the excess link bandwidth** to the \mathbf{r}^* . This is done by solving the LP model of the initial phase (Set 3 = *null*) while substituting the \mathbf{r}_{min} in Set 2 by the \mathbf{r}^* (defined in Step 6).

To solve the LP models, we use the classical Simplex method or the primal cutting-plane algorithm (when the bandwidth is quantized) [6].

The main difference in philosophy of this procedure from the previous one (section 2), is the proper selection of bandwidth levels and the increase of all connection rates simultaneously, level by level, instead of increasing, level by level, the rates of that calls (in priority) with the smallest bandwidth requirement. The proposed new algorithm is described in detail in the flow chart of Fig. 1.

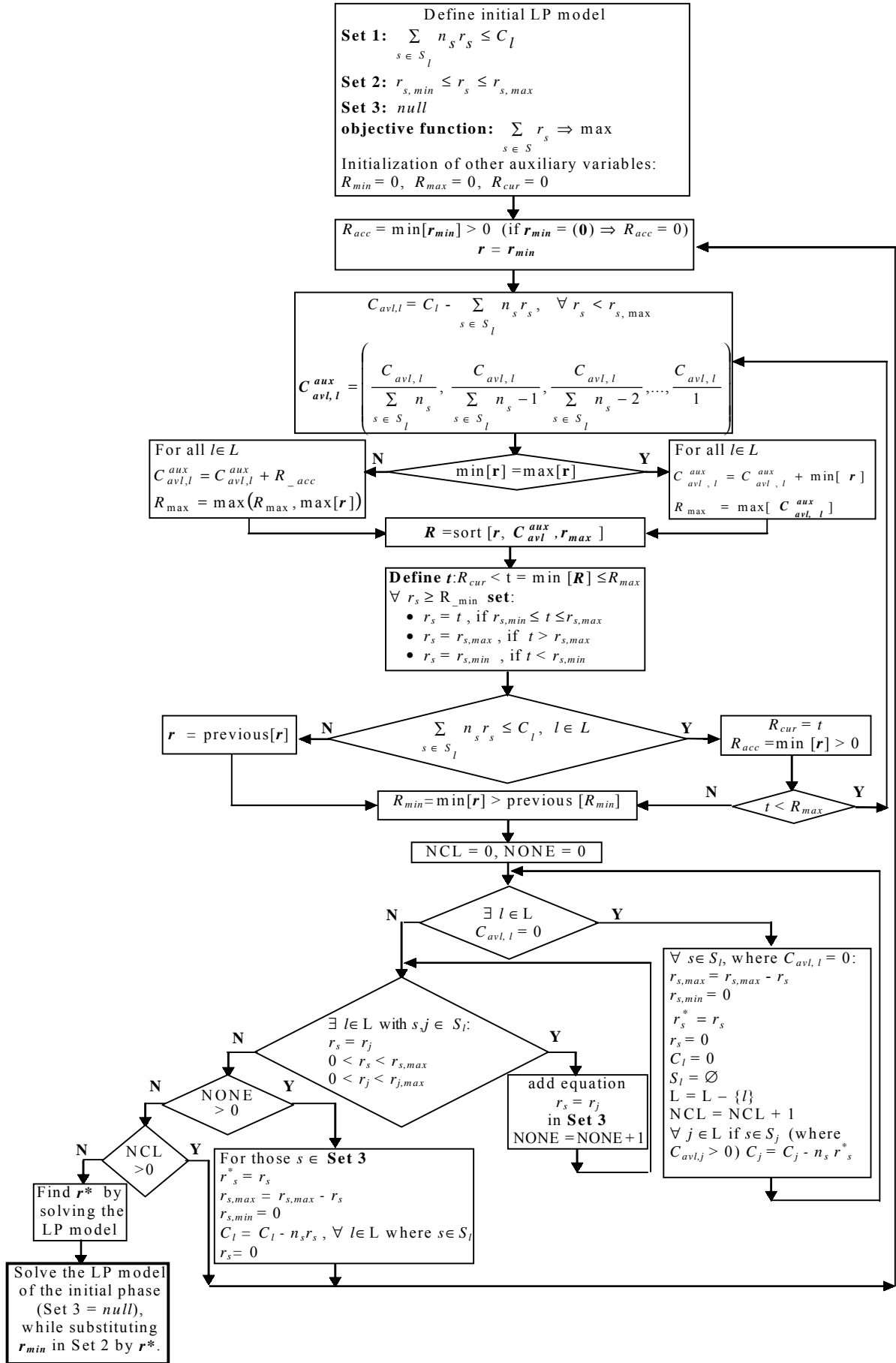


Fig. 1: The flow chart for the LP implementation of the GMM policy.

4. Numerical Examples

We consider the testbed network of Fig. 2, which has been adopted by the ATM Forum for performance comparison of various fairness criteria [7], and apply on it the proposed new algorithm, in order to define the GMM bandwidth allocation among the call connections. As Fig. 2 shows, the testbed network accommodates six calls: A, B, ..., F. Table 1 shows the number of connections, n_s , that each call consists of, as well as the minimum and maximum bandwidth requirements of each calls. More precisely, we consider three pairs of vectors (r_{min} , r_{max}), which are shown in Table 1, as cases 1, 2 and 3.

The constraint sets of the Initial phase of LP models:

$$\left. \begin{array}{l} 3r_A + 6r_D \leq 50 \\ 3r_A + 3r_B + 2r_F \leq 150 \\ 3r_A + 3r_B + 3r_C \leq 150 \\ 3r_B + 6r_E \leq 100 \end{array} \right\} \text{Set 1}$$

$$\left. \begin{array}{lll} \text{Case 1} & \text{Case 2} & \text{Case 3} \\ 4 \leq r_A \leq 34 & 2 \leq r_A \leq 12 & 2 \leq r_A \leq 7 \\ 7 \leq r_B \leq 22 & 4 \leq r_B \leq 18 & 4 \leq r_B \leq 10 \\ 24 \leq r_C \leq 35 & 12 \leq r_C \leq 20 & 12 \leq r_C \leq 20 \\ 5 \leq r_D \leq 9 & 1 \leq r_D \leq 5 & 1 \leq r_D \leq 5 \\ 8 \leq r_E \leq 24 & 3 \leq r_E \leq 14 & 3 \leq r_E \leq 14 \\ 20 \leq r_F \leq 100 & 10 \leq r_F \leq 100 & 10 \leq r_F \leq 100 \end{array} \right\} \text{Set 2}$$

$$\text{null} \} \text{Set 3}$$

Sets 1, 3 are common to all LP models (cases 1, 2, 3).

Running the new GMM algorithm for each case, the following LP models (Final phase) are formulated.

The constraint sets of the Final phase of LP models:

Set 1 is the same with that of the Initial phase.

$$\left. \begin{array}{lll} \text{Case 1} & \text{Case 2} & \text{Case 3} \\ 5.56 \leq r_A \leq 34 & 6.67 \leq r_A \leq 12 & 6.67 \leq r_A \leq 7 \\ 11.11 \leq r_B \leq 22 & 6.67 \leq r_B \leq 18 & 6.67 \leq r_B \leq 10 \\ 24 \leq r_C \leq 35 & 12 \leq r_C \leq 20 & 12 \leq r_C \leq 20 \\ 5.56 \leq r_D \leq 9 & 5 \leq r_D \leq 5 & 5 \leq r_D \leq 5 \\ 11.11 \leq r_E \leq 24 & 6.67 \leq r_E \leq 14 & 6.67 \leq r_E \leq 14 \\ 24 \leq r_F \leq 100 & 12 \leq r_F \leq 100 & 12 \leq r_F \leq 100 \end{array} \right\} \text{Set 2}$$

$$\text{null} \quad r_B = r_E \quad r_B = r_E \} \text{Set 3}$$

Since for some $s \in S$, the $r_{s,min}$ shown in Set 2 of the Final phase of the LP models is the final allocated rate to the call s , because of link saturation (e.g. case 1: $r_{D,min} = 5.56$) or because of $r_{s,min} = r_{s,max}$ (e.g. case 2: $r_{D,min} = 5$), the above LP models can be simplified. This simplification is taken in to consideration by the proposed new algorithm. However, we keep their above form for presentation purposes. Table 2 gives the GMM solution r^* of the LP model for each case, as well as the final allocated bandwidth r^* . The LP solution results (in all cases) by the well-known Simplex method.

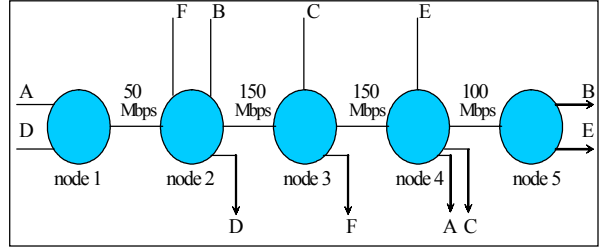


Fig. 2: Generic fairness configuration

Table 1: Elastic traffic parameters for calls A to F.

Call	n_s	r_{min} (Mbps)			r_{max} (Mbps)		
		Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
A	3	4	2	2	34	12	7
B	3	7	4	4	22	18	10
C	3	24	12	12	35	20	20
D	6	5	1	1	9	5	5
E	6	8	3	3	24	14	14
F	2	20	10	10	100	100	100

Table 2: GMM bandwidth allocation (the solution)

Call	Case 1		Case 2		Case 2	
	r^*	final r^*	r^*	final r^*	r^*	final r^*
A	5.56	5.56	6.67	6.67	6.67	6.67
B	11.11	11.11	11.11	11.11	10.0	10.0
C	33.33	33.33	20.0	20.0	20.0	20.0
D	5.56	5.56	5.00	5.00	5.00	5.00
E	11.11	11.11	11.11	11.11	10.0	11.67
F	50.00	50.00	48.33	48.33	50.0	50.0

5. Conclusion

We present the new GMM algorithm in procedural steps to describe its philosophy and in a flow chart to be directly convertible into software. Moreover we support it mathematically based on LP models.

References

- [1] I. Moscholios, M. Logothetis, "Call-level QoS assessment in ATM networks supporting elastic traffic", Proc. ICC2001, Helsinki, June 2001.
- [2] D. Bertsekas, R.Gallagher, "Data Networks", New Jersey, Prentice Hall, 1992.
- [3] Y.T. Hou, Bo Li, S.S. Panwar and H. Tzeng, "On network bandwidth allocation policies and feedback control algorithms for packet networks", Computer Networks 34 (2000), pp. 481-501.
- [4] I. Moscholios, M. Logothetis and G. Kokkinakis, "Evaluation of fair bandwidth allocation policies for elastic traffic", ACTA Tehnica Napocensis Electronics and Telecommunications Journal Vol. 43, No. 2, pp. 12-19, November 2002.
- [5] W. Ogryczak, T. Sliwinski and A. Wierzbicki, "Fair resource allocation schemes and network dimensioning problems", Journal of Telecommunications & Information Technology, pp. 34-42, 3/2003.
- [6] H.M.Wagner. "Principles of Operations Research", Prentice-Hall: Englewood Cliffs, New Jersey 1969.
- [7] R. Simcoe, "Test Configurations for fairness and other tests", ATM Forum/ 94-0557, July 1994.