Scheduling Policies for Two-State Smart-Home Appliances in Dynamic Electricity Pricing Environments

John S. Vardakas\textsuperscript{a,\ast}, Nizar Zorba\textsuperscript{b}, Christos V. Verikoukis\textsuperscript{c}

\textsuperscript{a}Iquadrat, Barcelona, Spain
\textsuperscript{b}QMIC, Al-Doha, Qatar
\textsuperscript{c}Telecommunications Technological Centre of Catalonia (CTTC), Barcelona, Spain

Abstract

In this paper we present and analyze online and offline scheduling models for the determination of the total power consumption in a smart grid environment. The proposed load models consider that each consumer’s residence is equipped with a certain number of appliances of different power demands and different operational times, while the appliances’ feature of alternating between ON and OFF states is also incorporated. Each load model is correlated with a scheduling policy that aims to the reduction of the power consumption through the compression of power demands or the postponement of power requests. Furthermore, we associate each load model with a proper dynamic pricing process in order to provide consumers with incentives to contribute to the overall power consumption reduction. The evaluation of the load models through simulation reveals the consistency and the accuracy of the proposed analysis.

Keywords: smart grid, power demand, dynamic pricing, performance

\ast Corresponding author

Email addresses: jvardakas@iquadrat.com (John S. Vardakas), nizarz@qmic.com (Nizar Zorba), cveri@cttc.es (Christos V. Verikoukis)
1. Introduction

The smart grid is a visionary power system that aims to the transformation of the existing infrastructure to a more consistent, efficient and user-centric power grid (1). The evolution to this next generation grid is based on the utilization of intelligent controllers and two-way communication between consumers and power utilities, with the aim of providing reliable and cost-effective energy supply (2). The communication technologies are vital to the future smart grid, as they provide the means for a synergic effort by both parties to manage the energy distribution and consumption, while limiting the cost and the environmental impact of the power generation (3). This inclination towards a Demand Side Management (DSM) promotes the active participation of consumers to compensate the imbalance of generation and demand, through the change of their power usage preferences (4).

Demand Response (DR) is one of the main DSM activities that aims to provide consumers with motives to change their electric use, in response to changes in electricity prices, as stated by the US Department of Energy (5). Designing an efficient DR program is an important issue that should target on the modification of the consumers’ demand profile. This objective can be realized by either reducing the power consumption of specific loads (6) or controlling the activation time of the requested load (7). In the former case, power scheduling programs regulate the operation of various appliances, in order to consume less power during system stress periods without affecting their functionality (e.g. air-conditions that can be adjusted to 25°C, in-
stead of $22^\circ\text{C}$, during a summer day) (8). Alternatively, in task scheduling programs requested loads that are able to be adjusted (e.g. water-heaters or laundry pairs) are shifted to off-peak hours (9). The decisions related to both scheduling methods (amount of power reduction or duration of the request delay) are taken either offline, based on past or predicted consumption patterns (10), or online, based on real-time observations of the power consumption (11).

The performance of a scheduling program can be further improved by applying an efficient pricing management scheme. A dynamic pricing model provides consumers with incentives to reduce their loads or to shift their power consumption to off-peak hours (12). At the same time, power utilities receive significant benefits from the application of dynamic pricing, since the reduced power consumption in high-peak hours de-escalates the need to activate expensive-to-run power plants (13). Moreover, the key features of the applied pricing model should be defined based on the applied scheduling policy, in order to provide a more rational charging policy to customers.

The performance evaluation of either power or task scheduling policies has been extensively studied in the literature, mainly through simulation (9), (14) or optimization methods (15), (16), (17). However, only a few analytical models have been proposed; power demand control policies are proposed and analyzed in (18) and (19). They take the current power consumption pattern into consideration so as to immediately activate or postpone a power request. In addition, a number of analytical models have been developed for the performance evaluation of electric vehicle charging infrastructures (20), (21). In all cases, the power requirement of each power request equals to 1
power unit, which is constant for the entire operating time of the device.

The contribution of this paper can be summarized in the following 5 points:

• We present the baseline policy and we develop an analytical model for the determination of the upper bound of the total power consumption in a residential area.

• We study both the power and task scheduling policies by presenting and analyzing two offline and two online scheduling policies that aim to reduce the total power consumption in the under study residential area. The baseline policy and the scheduling policies are applied to a residential area, where each residence is equipped with a specific number of appliances with different power demands, operational times and arrival rates of power requests.

• Moreover, we assume the more realistic case, where the power requirements of a number of appliances are not constant for their entire operation period. On the contrary, we consider that an appliance transits between two states of operation; an ON state, where the appliance operates under its nominal power and an OFF state where no power consumption occurs. The parameters related to the duration of ON and OFF periods and the transition probability from ON to OFF state and vice versa are also incorporated in our analysis.

• We also associate the proposed scheduling policies with a proper dynamic pricing scheme by providing different cost functions for each policy and we present analytical results for the total average cost.
Finally, we compare analytical results from the proposed scenarios with corresponding analytical results from (19). We demonstrate that the proposed analytical models achieve better performance regarding the total power consumption, while being more realistic since they consider multiple power requests of ON-OFF type with diverse power requirements.

The remainder of the paper is organized as follows. In Section 2 we introduce the analysis for the baseline policy and for the two proposed offline power demand policies, while in Section 3 we present and analyze the two proposed online power demand policies. Section 4 investigates the effect of a dynamic electricity pricing function to the total average cost for the proposed power demand policies. In Section 5 we evaluate the proposed analysis by comparing analytical and simulation results. The conclusions of our paper are stated in Section 6.

2. Offline Scheduling Policies

The following analysis is used as the baseline for the determination of the upper bound of the total power consumption in the under study residential area. We consider a residential area where each residence is equipped with up to $M$ appliances. All appliances in a residence are connected to an Energy Consumption Controller (ECC), while the ECC is connected to the Central Load Controller (CLC) through a Local Area Network (LAN), as depicted in Fig. 1. Each appliance alternates between ON and OFF periods. More precisely, when a type-$m$ appliance ($m = 1, \ldots, M$) is activated, it requires $r_m$ power units (p.u.) from the CLC. After this ON demand period,
appliance will either start an OFF period, without requiring any p.u., or terminate its operation. It should be noted that there is an explicit difference between an OFF state and the termination of the appliance’s operation: in the former case, after an OFF period the appliance returns to an ON period with probability 1, while in the latter case the appliance is switched off, as depicted in Fig. 2. An example of such an appliance is the air conditioner: when the user turns the appliance on, an ON period begins; when the desirable temperature is achieved, the appliance starts an OFF period until it is automatically re-activated when the temperature exceeds the preferable levels. When the user turns off the air-conditioner, the operation of the appliance is terminated.

Based on the aforementioned example, when the appliance attempts to return from the OFF state to the ON state, it re-requests $r_m$ p.u., so that a new ON period begins. We consider that after an ON demand period, a type-$m$ appliance is transferred to an OFF demand period with probability $a_m$, while it terminates its operation with probability $(1 - a_m)$. This probability $a_m$ is equal to zero for appliances that do not alter between ON and OFF demand periods, but they continuously require a constant amount of power units (equal to $r_m$).

The residential area under study is characterized by the parameter $P$, which denotes the maximum number of p.u. that the energy provider can support. When a consumer wishes to activate an appliance, a new power request arrives to the CLC from the consumer’s ECC. The CLC receives the request through a load control message that is sent in the LAN control channel. The arrival process of requests for type-$m$ appliances follows a
Poisson distribution, with a mean arrival rate denoted as $\lambda_m$. The Poisson distribution has been considered as a suitable solution for the modelling of the power requests’ arrival process (19), (22), (23).

When the appliance is transferred to an OFF demand period, it does not require power for its operation; therefore, in order to model this state, we assume that during the OFF period an identical fictitious appliance consumes $r_m$ p.u. in a fictitious power system, which is characterized by a fictitious maximum number $P_{\text{fict}}$ of supported p.u.. This fictitious power system is assumed, in order to model the transition of the appliance’s operation between ON and OFF power demand periods and to define the number of appliances that are present in an OFF state. Based on the aforementioned analysis, we define two states for each appliance:

1. state $i = 1$, for appliances in an ON power demand period,
2. state $i = 2$ for appliances in an OFF power demand period.

Therefore, when an appliance is in the OFF state, it consumes zero p.u. in the real power system, while a fictitious appliance is simultaneously present in the fictitious power system, where it consumes $r_m$ fictitious p.u. Furthermore, the ON and OFF demand periods are exponentially distributed with mean $d_{i,m}^{-1}$.

We consider that the number of type-$m$ appliances that are active and consume $r_m$ p.u. is denoted as $n_m^1$, while the number of type-$m$ fictitious appliances that are active in the fictitious power system is denoted as $n_m^2$. Based on these definitions, the total number of p.u. that are consumed in the real and the fictitious power system are respectively given by:
where \( j_s \) is the total number of p.u. in the real \((s = 1)\) and the fictitious \((s = 2)\) power system. The CLC checks the values of both \( j_1 \) and \( j_2 \) in order to accept a new power request by considering both the following conditions:

\[
\begin{align*}
    j_1 + r_m &\leq P \\
    j_1 + j_2 + r_m &\leq P_{fict}
\end{align*}
\]  

These conditions of transition are illustrated in Fig. 2. The first condition ensures that the total number of p.u. in use will not exceed the maximum number \( P \) of p.u. that the energy provider can support. The second condition prevents the CLC from accepting new power requests, when a large number of appliances are in the OFF power demand state.

The distribution of occupied p.u. \( \vec{j} = (j_1, j_2) \), denoted as \( q(\vec{j}) \), is given by the following two-dimensional recursive formula, which has been originally proposed for the distribution of the occupied bandwidth in multi-rate communication networks (24):

\[
\sum_{i=1}^{2} \sum_{m=1}^{M} r_{i,m,s} \cdot f_{i,m}(\vec{j}) \cdot q(\vec{j} - R_{i,m}) = j_s \cdot q(\vec{j})
\]  

where the parameter \( i \) indicates the state of the appliance and the parameter \( s \) denotes the power system (real or fictitious). Also,

\[
r_{i,m,s} = \begin{cases} 
    r_m & \text{if } i = s \\
    0 & \text{if } i \neq s
\end{cases}
\]  

which denotes that when the appliance is, for example, at the ON state \((i = 1)\), then it requires \( r_m \) p.u. only in the real system \((s = 1)\) and not in
the fictitious one \((s = 2)\). Furthermore, \(R_{i,m}\) is a \(2 \times 2\) matrix with elements \((r_{i,m,1}, r_{i,m,2})\) and \(f_{i,m}\) is a function related to the basic characteristics of each type-\(m\) appliance:

\[
f_{i,m} = \begin{cases} \frac{\lambda_m}{d_{1,m}(1-a_m)} & \text{if } i = 1 \\ \frac{\lambda_m a_m}{d_{2,m}(1-a_m)} & \text{if } i = 2 \end{cases}
\]

(5)

It should be noted that Eq. (3) can be applied in the smart grid case, since it also assumes Poisson arrivals and generally distributed service-times. Furthermore, the assumption of discrete power consumption can provide efficient results, especially when 1 p.u. is considered equivalent to a very small value of the continuous power consumption, without significantly increasing the computation complexity, since Eq. (3) is a recursive formula.

Due to the finite nature of \(P\), there is a probability \(B_m\) that after the acceptance of a power request the total number of p.u. exceeds \(P\). From Eq. (3), \(B_m\) can be calculated as the sum of the probabilities of all states that are defined by Eq. (2) and make the total number of p.u. in use to exceed \(P\):

\[
B_m = \sum_{\{j\in r_{1,m,1}+j_1>P\cup [r_{1,m,1}+j_1+j_2>P^{\text{fict}}]\}} \left(\frac{q(j)}{Q}\right)
\]

(6)

where \(Q\) is the normalization constant \(Q = \sum_{j_1=0}^{P} \sum_{j_2=0}^{P^{\text{fict}}-j_1} q(j_1, j_2)\). Furthermore, if \(P < P^{\text{fict}}\), there is a probability that when an appliance is switched off and tries to re-activate, the available p.u. in the system is less than the power requirements of the appliance. This re-activation blocking probability can be calculated by the normalized sum of the rates from all states \(\{(P - r_m + 1 \leq j_1 \leq P) \cap (j_1 + j_2 \leq P^{\text{fict}})\}\) where re-activation blocking occurs, over the rates from all possible states:
\[
C^*_m = \frac{\sum \{ \tilde{j} \mid (P - r_m + 1 \leq j_1 \leq P) \cap (j_1 + j_2 \leq P_fict) \} \ y_{2,m}(\tilde{j}) q(\tilde{j}) d_{2,m}}{\sum \{ \tilde{j} \mid (j_1 \leq P) \cap (j_1 + j_2 \leq P_fict) \} \ y_{2,m}(\tilde{j}) q(\tilde{j}) d_{2,m}}
\] (7)

Note that if \( P = P_fict \), then \( C^*_m = 0 \); therefore appliances can always return to the ON state. Equations (6) and (7) can be used for the determination of the minimum value of \( P \) which guarantees that the requested p.u. do not suffer an outage probability (for a new power request or for a re-activation request), not larger than a predefined maximum value \( e \).

In the following subsections we present the analyses for the two proposed offline scheduling policies that aim to reduce the total power consumption in the residential area under study: the Offline Power Scheduling Policy (OFF-PSP) and the Offline Task Scheduling Policy (OFF-TSP). The analysis of the baseline for the offline scheduling policies also applies as the baseline for the online scheduling policies that are presented in Section 3.

2.1. The OFF-PSP

The application of the OFF-PSP is based on the knowledge of the power consumption of the referred residential area. The system administrator considers either past power consumption patterns from the same area and the same time period or prediction models that extract the consumption profile of the same region. Based on this information, the administrator makes decisions on the power demand scheduling, in order to reduce the total power consumption in peak demand periods. Under the OFF-PSP, these decisions define an energy-based scheduling, where appliances are prompt to gradually reduce their power demands when the power consumption is expected to exceed predefined thresholds. The utilization of a set of power thresholds aims
to minimize the effect of a forcible power reduction, so that consumers see marginal decrease in convenience and comfort.

The determination of the power threshold set is a vital issue for the successful implementation of the scheduling policy. An example of the designation of a 4 power threshold set is illustrated in Fig. 3. Under the OFF-PSP, for a time period where the power consumption is expected to exceed the first power threshold, users are prompt that the power demands of specific appliances will be compressed by a factor of \( \omega_1^{m} \), with \( 1 < \omega_1^{m} < 1 \), so that the new power demand is \( (1-\omega_1^{m}) \cdot r_m \). In order to guarantee the proper operation of the appliance under this compressed power demand, the appliance’s operational times (both in ON and OFF states) are increased by a factor \( \xi_1^{m} \), with \( 1 < \xi_1^{m} < 1 \), so that the new operational times are \( (1+\xi_1^{m})d_i^{m} \). By considering a set of \( T \) thresholds, these factors are defined so that power demands are decreasing and operational times are increasing with the increase of power consumption, i.e. \( \omega_1^{m} > \ldots \omega_t^{m} > \ldots > \omega_T^{m} \) and \( \xi_1^{m} < \ldots \xi_t^{m} < \ldots < \xi_T^{m} \), with \( t = 1, \ldots, T \). These factors are equal to zero for appliances that are not able to compress their power demands.

The determination of the \( P \) minimum value is achieved by following a procedure that is based on the analysis for the baseline policy. More specifically, when the total power consumption is expected to be between thresholds \( P_{t-1} \) and \( P_t \), the distribution \( q_t(j) \) of the number of p.u. in use can be calculated through Eq. (3), by using the corresponding values for the power demands and operational times. Furthermore, for each distribution \( q_t(j) \) the outage and the re-activation probabilities can be determined through Eq. (6) and Eq. (7), respectively, where the parameter \( r_m \) is replaced by the
compressed power demand \((1 - \omega_m^t) \cdot r_m\). These values can be used to determine the minimum value of \(P\) which guarantees that a new power request (or a re-activation request) will not suffer an outage probability larger than a predefined maximum value \(e\).

2.2. The OFF-TSP

The OFF-TSP considers the same operational principles of the OFF-PSP; the load scheduling is based on past or predicted consumption patterns and on the definition of a power threshold set. However, when the total power consumption is expected to be above a threshold \(P^t\), power demands are not compressed; instead, power requests are delayed for a specific time period, which is different for each appliance. This postponement is achieved by utilizing \(M\) buffers (one for each type of appliance) that are installed in the CLC. Power requests that arrive at the CLC when the total power consumption is expected to be between thresholds \(P^t - 1\) and \(P^t\) are delayed for a constant period \(\delta_{m}^{t}\), with \(\delta_{m}^{1} < ... < \delta_{m}^{T-1} < \delta_{m}^{T}\); therefore, the delay builds up when the total power consumption increases. After this delay, power requests immediately attempt to access the system. The values of the parameters \(\delta_{m}^{t}\) are equal to zero for appliances that cannot tolerate delays.

The \(P\) minimum value determination is achieved by considering the analysis for the baseline policy, as in the case of the OFF-PSP. The main difference between the analysis of OFF-TSP and for the baseline policy is that the arrival rate \(\lambda_m\) of power requests for type-\(m\) appliances in the baseline policy is larger than the arrival rate of requests in the OFF-TSP, due to the delay in the buffers. By considering the delay \(\delta_{m}^{t}\) and the arrival rate \(\lambda_m\) of power requests to the buffers, the new arrival rate of power requests at the
controller is formulated to:

\[ \lambda^t_m = \left( \delta^t_m + 1/\lambda_m \right)^{-1}, \ t = 1, ..., T \]  \hspace{1cm} (8)

since the time $1/\lambda^t_m$ between two consecutive power request arrivals must also consider the delay $\delta^t_m$ at the buffer. The new arrival rates define new values for the functions $f_{i,m}$, which can be calculated through Eq. (5). Furthermore, the distribution $q_t(\vec{j})$ of the number of p.u. in use when the total power consumption is expected to be between thresholds $P^{t-1}$ and $P^t$ can be calculated through Eq. (3) by using the corresponding values for $f_{i,m}$. Finally, as in the case of OFF-PSP, the outage and the re-activation probabilities can be determined through Eq. (6) and Eq. (7), respectively, while these values can be used for the determination of the minimum value of $P$, so that a new power request (or a re-activation request) will not suffer an outage probability larger than a predefined maximum value $e$.

3. Online Scheduling Policies

In contrast to the offline procedures that consider past or predicted consumption patterns, the online power demand policies make decisions for the compression or the delay of power demands based on the current power consumption. This is achieved by monitoring the ongoing power consumption and by setting the amount of power compression (or delay) based on power thresholds. In the following subsections we present the proposed analysis for the Online Power Scheduling Policy (ON-PSP) and the Online Task Scheduling Policy (ON-TSP).
3.1. The ON-PSP

In this scheme we assume that a number $H$ of thresholds is defined for the total power consumption: these thresholds are similar to the offline policies’ thresholds. Upon the arrival of a new power demand request for a type-$m$ appliance, if the total power consumption is less than the first threshold $P_{h=1}^{\text{real}}$, the power request is accepted with its initial requirements $r^0_m = r_m$ and $(d_{i,m}^0)^{-1} = d_{i,m}^{-1}$. If the total power consumption exceeds the first threshold $P_{h=1}^{\text{real}}$, then the power request is accepted with a compressed power demand $r_{m}^{h=1}$, with $r_{m}^{h=1} < r_m$. In order to guarantee the proper operation of the appliance under a compressed power demand, we assume that its operational time is increased, i.e. $(d_{i,m}^{h=1})^{-1} > d_{i,m}$. The new power request may attempt $H+1$ times to be accepted, each time with a reduced power requirement $r_{m}^{h}$, ($h = 1, \ldots, H$) and an increased operational time $d_{i,m}^{h}$; the pair $(r_{m}^{h}, d_{i,m}^{h})$ is applied when the total number $j_1$ of occupied p.u. is $P_{h}^{\text{real}} < j \leq P_{h+1}^{\text{real}}$, with $P_{H+1}^{\text{real}} = P$. After attempting $H+1$ times, the power request will be accepted but the total power consumption will exceed the maximum supported number $P$ of p.u.. Similarly, a set of $H$ thresholds is defined for the fictitious system, where $P_{h}^{\text{fict}}$ is the fictitious threshold that corresponds to the real threshold $P_{h}^{\text{real}}$. The application of multiple thresholds results in a gradual reduction of the requested power when the total power consumption increases, which in turn has a smaller effect on the consumers’ convenience. It should also be noted that this compression mode is applied only to appliances that are capable of reducing their power demands and at the same time extend their operation time, e.g. air conditions or water heaters.

The following approximate recursive formula is proposed for the determin-
nation of the distribution of the occupied p.u. under the PSP:

\[
\sum_{i=1}^{2} \sum_{m=1}^{M} f_{i,m} r_{i,m,s} \varphi_m(\vec{j}) q(\vec{j} - R_{i,m}) + \\
+ \sum_{i=1}^{2} \sum_{m=1}^{M} \sum_{h=1}^{H} f_{i,m}^{h} r_{i,m,s}^{h} \gamma_{m}^{h}(\vec{j}) q(\vec{j} - R_{i,m}^{h}) = j_s q(\vec{j})
\]  

(9)

where \( R_{i,m}^{h} \) is a \( 2M \times 2 \) matrix with elements \((r_{i,m,1}^{h}, r_{i,m,2}^{h})\). The parameters \( f_{i,m} \) are given by Eq. (5), while the parameters \( f_{i,m}^{h} \) are similarly defined as:

\[
f_{i,m}^{h} = \begin{cases} 
\frac{\lambda_m}{d_{1,m}(1-a_m)} & \text{if } i = 1 \\
\frac{\lambda_m a_m}{d_{2,m}(1-a_m)} & \text{if } i = 2, \ h = 1, \ldots, H
\end{cases}
\]  

(10)

The function \( \varphi_m(\vec{j}) \) is used in order to control the recursive formula, based on whether a type-\( m \) appliance is capable of reducing its power demands. Also, the function \( \gamma_{m}^{h}(\vec{j}) \) is applied in order to define under which threshold the power request is accepted. These two functions are respectively given by:

\[
\varphi_m(\vec{j}) = \begin{cases} 
1 & \text{if } \left[ (j_1 \leq P_{real}^{1} + r_{1,m}) \cap (j_1 + j_2 \leq P_{fict}^{1} + r_{1,m}) (r_{1,m} > 0) \right] \\
\cup \left[ (j_1 \leq P) \cap (r_{1,m} = 0) \right] \\
0 & \text{otherwise}
\end{cases}
\]  

(11)

\[
\gamma_{m}^{h}(\vec{j}) = \begin{cases} 
1 & \text{if } \left[ (P_{h}^{real} + r_{1,m}^{h} > j_1 \geq P_{h-1}^{real} + r_{1,m}^{h}) \cap \\
(P_{h}^{fict} + r_{1,m}^{h} > j_1 + j_2 \geq P_{h-1}^{fict} + r_{1,m}^{h}) \cap (r_{1,m}^{h} > 0) \right] \\
0 & \text{otherwise}
\end{cases}
\]  

(12)

The function \( \varphi_m(\vec{j}) \) considers that if a type-\( m \) appliance is able to compress its power demand \((r_{1,m}^{h} > 0)\), then it will be accepted with its initial demand only when \((j_1 \leq P_{1}^{real} + r_{1,m})\) and \((j_1 + j_2 \leq P_{1}^{fict} + r_{1,m})\), while
for appliances that cannot suffer power compression \((r^h_{1,m} = 0)\), their requests are accepted in any system state. Furthermore, the function \(\gamma^h_{m}(\vec{j})\) activates the second term of the left hand side of Eq. (9) only for appliances that are able to compress their demands \((r^h_{1,m} > 0)\) and only when the total number \(j_1\) of the p.u. in use is between two consecutive thresholds \((P^\text{real}_h + r^h_{1,m} > j_1 > P^\text{real}_{h-1} + r^h_{1,m})\) and \((P^\text{fict}_h + r^h_{1,m} > j_1 + j_2 > P^\text{fict}_{h-1} + r^h_{1,m})\).

The proof of Eq. (9) is provided in Appendix, where firstly a recursive formula for the case of a single threshold is derived. This formula is then extended in order to cover the case of multiple thresholds, while the necessary approximations are also presented. Equation (9) can be used for the calculation of the outage probabilities for both groups of appliance types: for the appliances that are not able to compress their power demands the outage probability \(B_m\) is calculated through Eq. (6), whereas, in case of appliances that are capable of reducing their demands, the outage probability \(B_{m1}\) is calculated by considering their smallest power demand \(r^h_{1,m}:\)

\[
B_{m1} = \sum_{\{j| [r_{1,m} + j_1 > \text{P}v(j_{1,m} + j_1 + j_2 > \text{P}fict)\}} \left( q(\vec{j})/Q \right) \tag{13}
\]

Furthermore, the re-activation blocking probability \(C_m\) of appliances that are not able to compress their power demands under the PSP is given by Eq. (7). For appliances that compress their demands, the re-activation blocking probability can be calculated by the following proposed equation:

\[
C_{m1} = \frac{\sum_{h=0}^{H} \sum_{t=0}^{T} \{j| (P - r^t_{m} + 1 \leq j_1 \leq P) \cap (j_1 + j_2 \leq P^fict)\}}{\sum_{t=0}^{T} \{j| (j_1 \leq P) \cap (j_1 + j_2 \leq P^fict)\}} \cdot y^2_{3,m}(\vec{j})q(\vec{j})d^t_{2,m} \tag{14}
\]
which is derived by the normalized sum of the rates from all states, where re-activation blocking occurs, over the rates from all possible states (as in the case of Eq. (7)), but for the entire set of $H$ thresholds.

The values of both sets ($B_m$, $B_{m1}$) and ($C_m$, $C_{m1}$) are used in order to calculate the minimum number $P$ of the total p.u., so that the outage probability (for a new power request or for a re-activation request) will not exceed a predefined value $e$.

3.2. The ON-TSP

In the ON-TSP the same set of thresholds is used as in the ON-PSP. However, in the case of the ON-TSP, the appliances’ power requirements are not compressed when the total power consumption exceeds a threshold. Instead, power requests are delayed in $M$ buffers of finite length (one buffer for each type of appliance) that are installed in the CLC. After this delay, power requests instantly attempt to access the system. In this way, the resulting arrival rate of the power requests is reduced, since the time between two consecutive arrivals is increased by the delay in the buffer. Therefore, the reduced arrival rate results in a lower probability of reaching high power consumption states. The delay duration for each appliance type varies based on the total power consumption and the applied thresholds. By considering that $H$ thresholds are applied when the total power consumption $j_1$ is $P_{h-1}^{\text{real}} < j_1 \leq P_{h}^{\text{real}}$, the delay duration is defined as $\delta_m^h$, with $\delta_m^1 < ... < \delta_m^{H-1} < \delta_m^H$; therefore, the delay increases with the increase of the total power consumption.

The buffers’ utilization for the power requests’ delay affects the resulting request arrival rate, since the time between two consecutive arrivals is in-
creased. Similar to OFF-TSP, for a type-$m$ appliance this inter-arrival time is equal to the sum of the inter-arrival time $1/\lambda_m$ of requests that arrive at the buffer plus the time $\delta^h_m$ that the request is delayed at the buffer itself. The inverse of this sum defines the arrival rate of power requests for type-$m$ appliances at the controller and can be calculated by using Eq. (8). By using the new arrival rate values and based on Eq. (5), the functions $f^h_{i,m}$ that refer to power requests that arrive at the controller after the buffers are defined as:

$$f^h_{i,m} = \begin{cases} \frac{\lambda^h_m}{\lambda_m + (1-a_m)} & \text{if } i = 1 \\ \frac{\lambda^h_m}{\lambda_m - a_m} & \text{if } i = 2 \end{cases}, \quad h = 0, ..., H \tag{15}$$

where $\lambda^0_m = \lambda_m$. Equation (15) indicates that the function $f^h_{i,m}$ is state dependent, since it is determined by the number of p.u. in use; therefore $f^h_{i,m} \equiv f^h_{i,m}(\vec{j})$. By considering the set of $H$ thresholds, the local balance equations for the ON-TSP are the following:

$$f^0_{i,m}(\vec{j})q(\vec{j} - R_{i,m}) = y_{i,m}(\vec{j})q(\vec{j})$$

for $j_1 \leq P_1 + r_{i,m}$ and $j_1 + j_2 \leq P_1^{fict} + r_{i,m}$ \tag{16}

$$f^h_{i,m}(\vec{j})q(\vec{j} - R^h_{i,m}) = y^h_{i,m}(\vec{j})q(\vec{j})$$

for $P^{real}_{h-1} + r_{1,m} \leq j_1 < P^{real}_h + r_{1,m}$ and $P^{fict}_{h-1} + r_{1,m} \leq j_1 + j_2 \leq P^{fict}_h + r_{1,m}$ \tag{17}

The set of Eq. (16) and Eq. (17) cannot reformulate a recursive formula for the distribution of the p.u. in use. This is due to the state dependency of $f^h_{i,m}(\vec{j})$ that prevents the use of the procedure that is followed for the ON-PSP case in Appendix A. Therefore, the set of Eq. (16) and Eq. (17) can only be
solved by using a numerical method for the determination of \( q(\vec{j}) = q(j_1, j_2) \).

It should be noted that since \( 0 \leq j_1 \leq P \) and \( 0 \leq j_1 + j_2 \leq P_{fict} \), the computational complexity of the numerical solution is significant for large values of \( P \) and \( P_{fict} \), since by average, a number of \( (P \cdot P_{fict})/2 \) equations have to be solved for the derivation of the distribution of the p.u. in use. This fact proves the necessity of recursive formulas for the determination of similar distributions.

Finally, under the ON-TSP the probability that the total power consumption will exceed \( P \) upon the arrival of a power demand that requests \( r_m \) p.u. can be calculated through Eq. (6). This equation is used since in the ON-TSP case power demands are not compressed, as in the case of the baseline policy. Similarly, the re-activation blocking probability for the case of the ON-TSP is calculated through Eq. (7). It should be noted that if the delay that power requests suffer is set to zero for all types of appliances and for all thresholds, then the ON-TSP coincides with the baseline policy.

4. Dynamic Pricing Analysis

The application of a scheduling policy should be followed by the utilization of a smart pricing scheme, with the aim of providing incentives to customers that undergo power compression or delays. The design of the electricity pricing procedure should be carefully implemented, in order to ensure that the characteristics of the applied scheduling policy are abundantly considered. The definition of a pricing scheme for the proposed scheduling policies should result in a dynamic scheme, which considers the application of the power thresholds, and the amount of power compression and delays that
customers suffer. To this end, the definition of a cost function that considers
the current power consumption is vital, for the provision of a more rational
charging policy that remunerates the consumers for their involvement in the
applied scheduling policy.

Let \( C(j) \) be the cost function associated with the total number \( j \) of p.u.
in use. This cost function should be an increasing function, so that the
total power cost is increased by the increase of the power consumption. For
the case of the baseline policy, a simple increasing function in the form of
\( C(j) = a \cdot j^k \) can be defined, in order to define an upper bound for the
total power consumption. However, for both the power and task scheduling
policies, the cost function should consider the alternations of the appliances’
operation, due to the application of power thresholds. Therefore, the cost
function is a piecewise linear convex function and can be defined as:

\[
C_{sch}(j) = \begin{cases} 
  x_1 \cdot j^{n_1} & \text{if } j_1 \leq P_{real}^1 \\
  x_2 \cdot j^{n_2} & \text{if } P_{real}^1 < j_1 \leq P_{real}^2 \\
  \vdots & \\
  x_T \cdot j^{n_T} & \text{if } P_{real}^T < j_1 \leq P_{real}^T 
\end{cases}
\]

where \( x_1 \leq x_2 \leq \ldots \leq x_T \) and \( n_1 \leq n_2 \leq \ldots \leq n_T \). In this way, the electricity
cost is increased with the increase of the total power consumption, while
consumers are motivated to participate in a scheduling policy in order to
achieve savings on their electricity bills. The aforementioned cost function is
applied to both offline policies, while by substituting \( T \) with \( H \), it is applied
to both online policies. The parameters \( x_t \) and \( n_t \) (\( t = 1, \ldots, T \)) should be
determined as a function of the average reduction of the power demands of all
\( M \) types of appliances (for the OFF-PSP and ON-PSP cases) or the average
delay that power request suffer (for the OFF-TSP and OFF-TSP cases).

The cost function can be used in order to define the total average cost as:

$$A_{C(j_1)} = \sum_{j_1=0}^{p_{real}} j_1 \cdot q(\tilde{j}) \cdot C(j_1)$$  \hspace{1cm} (19)

The minimization of the total average cost, for a given set of power requests, can be achieved by alternating the distribution $q(\tilde{j})$, in order to obtain the minimum value of $P$. For the case of the power scheduling policies (PSP), this objective is attained by considering the maximum possible power demand reduction and the corresponding maximum possible expansion of the appliances’ operational times. The definition of these maximum limits is subject to the specific characteristics of each appliance type. On the other hand, for the task scheduling policies (TSP), the minimization of the total average cost can be achieved by considering the maximum permissible delay for the power requests that minimizes the final arrival rate of power requests. The decisions for applying this maximum delay limit should be taken with the consideration of the appliances’ features and the consumers’ tolerance for postponing the operation of their appliances.

5. Evaluation and Discussion

In this section the proposed analysis is evaluated through simulation. To this end, a simulator is built in order to provide a test environment of the residential area under study. The simulator assumes a large residential area (due to the Poisson assumption for the power requests), where each residence is equipped with $M=10$ appliances: 1) a laundry pair, 2) a water heater, 3) an electric stove 4) a dishwasher, 5) a refrigerator, 6) an air condition, 7) an
entertainment set, 8) a home office set, 9) lighting and 10) a Plug-in Hybrid Electric Vehicle (PHEV). We consider that the entertainment set, the home office set and lighting do not alter between ON and OFF states; therefore
\[ a_7 = a_8 = a_9 = 0 \] and
\[ d_{2,7}^{-1} = d_{2,8}^{-1} = d_{2,9}^{-1} = 0, \] since these appliances do not start an OFF period. The power demands of the appliances are
\[ r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9, r_{10} \] = (25, 40, 20, 9, 6, 25, 7, 5, 4, 50) p.u. per hour of operation. The values of the power demands are defined by considering the characteristic appliance power consumption (25) and by assuming that 1 p.u. = 100 Watt.

We consider 6 different power consumption cases, which are defined based on the daily average power consumption in an urban area (26): very low (case A), low (case B), medium (case C), high (case D), very high (case E) and extremely high (case F) power consumption periods. In each period, different appliances are activated, with different arrival rates and operational times. The types of appliances that are activated in each case and the corresponding parameters \( (\lambda_m, a_m, d_{1,m}^{-1}, d_{2,m}^{-1}) \) are listed in Table 1. We also consider that the fictitious power system has a fictitious maximum number \( P_{\text{fict}} \) of supported p.u. that is 10% larger than the maximum number \( P \) of supported p.u. in the real power system. For each case, we assume \( 10^6 \) power requests arrive at the CLC, while a stabilization time that corresponds to the first \( 5 \times 10^4 \) power requests is assumed, in order for the system to reach the steady state. Furthermore, both analytical and simulation results are obtained so that the probability that the total power consumption will not exceed \( P \) is below \( e = 10^{-5} \). In all cases, simulation results are obtained as mean values of 8 runs (each one of \( 10^6 \) power requests), with 95% confidence interval, while
only the mean values are used in the following figures, since the reliability ranges are found to be very small.

5.1. Evaluation of the offline scheduling policies

The evaluation of the two offline scheduling policies is realized through the comparison of the results for the minimum value of $P$, to the corresponding results of the baseline policy. To this end, we assume that three power thresholds are defined and in each power consumption case, the power consumption always exceeds these thresholds. The latter assumption is considered in order to illustrate the effect of the power demand compression and the power request delay on the total number of requested p.u. When the power consumption is expected to exceed the first threshold, for the OFF-PSP case we assume that power demands of only 5 types of appliances (laundry pair, water heater, electric stove, air condition and PHEV) are prompted to reduce their power demands by 5% and at the same time expand their operational times by 6%. For the OFF-TSP case, power requests from the same 5 types of appliances are delayed for 20 sec. In both cases, all other 5 types of appliances do not alter their demands and no power request delay occurs. Furthermore, when the power consumption exceeds the second and the third power threshold, for the OFF-PSP case the power demands are compressed by 9% and 10%, respectively, while the operational times are expanded by 14% and 15%, respectively. For the OFF-TSP case, power requests are delayed by 40 sec. and 60 sec., when the power consumption exceeds the second and the third power threshold, respectively. In both OFF-PSP and OFF-TSP, all other parameters have the same values as in the case of the baseline policy, in order to guarantee a fair comparison. Note that in the case of the OFF-
TSP, the delay by 20, 40 and 60 sec. results in a reduction of the power requests arrival rate from the average value of 1.35 requests/min. to 0.93 requests/min, 0.71 requests/min., and 0.6 requests/min., respectively.

In Fig. 4 and Fig. 5 we present analytical and simulation results for the minimum value of $P$ that the energy provider can support under the OFF-PSP and the OFF-TSP, respectively, and for the 6 power consumption cases. As the results reveal, the accuracy of the proposed analysis, is absolutely satisfactory. As it was anticipated, the power demand compression and the power request delay conclude in the decrease of the total number of requested p.u. More specifically, for the OFF-PSP case, the total number of requested p.u. is decreased in average by 8.71%, 14.3%, and 20.2%, for the three reductions of power demands, respectively, while for the OFF-TSP the numbers are 15.1%, 24.4%, and 30.8%, respectively.

We further study the performance of the two offline power demand policies by considering 3 predefined power thresholds, which are determined based on the results of the baseline policy. The 3 power thresholds are $P_1=350$ p.u., $P_2=450$ p.u. and $P_3=500$ p.u.. For every power consumption case, the compression of power demands and the expansion of the operational times for the OFF-PSP, as well as the power request delay for the OFF-TSP follow the same assumptions as the ones that are applied in Fig. 4 and Fig. 5. In Fig. 6 we comparatively present analytical results for the baseline policy, the OFF-PSP and the OFF-TSP for the 6 power consumption cases. We observe that the best performance is achieved under the OFF-TSP, since the average reduction of the power consumption is 22.5%, which is significant compared to the average reduction of 12.4% that is achieved under the OFF-PSP. Ev-
Identically, in both scheduling policies the power consumption reduction is due to the compressed power demands (for the OFF-PSP) and the delayed power requests (for the OFF-TSP), while the superiority of the OFF-TSP over the OFF-PSP is due to the selection of the delay and compression parameters; similar power consumption reduction is achieved for the two policies by increasing the delay parameters by 8%.

5.2. Evaluation of the online scheduling policies

For the evaluation of the online scheduling policies we assume that in both analysis and simulation three thresholds $P_1$, $P_2$ and $P_3$ are considered, with $P_1=40\%P$, $P_2=60\%P$ and $P_3=80\%P$. We consider two sets of parameters for the compressed power demands and the delayed power requests in the ON-PSP and ON-TSP, respectively. The first parameter set is the same as the one that is used in the case of the offline scheduling policies. For the second set of parameters, if the total number of p.u. in use exceeds the first threshold $P_1$, the 5 types of appliances compress their demands by 10% and at the same time they expand their operational times (both in ON and OFF states) by 12%. If the total number of p.u. in use exceeds the second threshold $P_2$, the same 5 types of appliances compress their demands by 15% and at the same time they expand their operational times by 16%, while for the third threshold these values are 20% and 22%, respectively. Furthermore, for the ON-TSP, power requests from the same aforementioned types of appliances are delayed for 45 sec., 70 sec. and 95 sec., when the total number of p.u. in use exceeds the first, the second and the third threshold, respectively. For a fair comparison between the offline and the online policies, we assume that no power thresholds are defined for the fictitious power system.
In Figs. 7 and 8 we present analytical and simulation results for the total number $P$ of p.u. that the energy provider can support, for the ON-PSP and ON-TSP, respectively, and for both the two set of parameters. The results show a similar decrease of the total power consumption that is observed in the offline scheduling policies. Moreover, for the first set of parameters, the online scheduling policies achieve slightly better performance compared to the baseline policy (average reduction of 1.7% and 2.4% for the ON-PSP and ON-TSP, respectively), but worse than the corresponding offline policies. This is because in the offline policies, the power compression and the request delay is performed for all power requests, regardless of the current power consumption. However, in the case of the online policies, a number of power requests are accepted with their initial requirements, when the current power consumption is below the first power threshold. In this way, a significant number of power requests do not suffer power compression or delay, therefore the total power consumption is higher than the consumption in the case of the offline policies. To this end, we considered the second set of parameters, in order to demonstrate the impact of the amount of power compression or the delay, to the total power consumption of the residential area under study. It should be noted that the computational time for the derivation of the results of the ON-TSP is significantly higher (25 minutes in average, using a quad core 2.53 GHz CPU and 4GB RAM), compared to computational time for the ON-PSP case (less than 1 sec.). Evidently, the superiority of the ON-PSP model, in terms of the computational complexity, is the result of the application of a recursive formula.

Next, we compare analytical results of the proposed scheduling policies
with corresponding results from the control policies of (19). These control policies, named Threshold Postponement (TP) and Control Release (CR) consider unit power requests (a single type of load) and constant power demands, without considering appliances of ON-OFF type. Since the characteristics of TP are similar to the proposed task scheduling policies, we compare the TP results only with the results of the OFF-TSP and ON-TSP. The following equivalences are considered, in order to provide a fair comparison:

\[
(d^*_m)^{-1} = \frac{a_m}{1-a_m} (d_{1,m}^{-1} + d_{2,m}^{-1}) + d_{1,m}^{-1} \quad (20)
\]

\[
(a/\mu) = \sum_{m=1}^{M} r_m \cdot (\lambda_m / d^*_m) \quad (21)
\]

The first equivalence is used in order to calculate the equivalent operational time of a type-\(m\) appliance that does not alternate between ON and OFF states, based on the parameters of the ON-OFF appliance. Equation (20) provides the average time that an ON-OFF appliance is initially activated, until its final shutdown and it includes both ON and OFF operation periods, by considering the mean number of transitions between the two states. The second equivalence derives the equivalent ratio of the arrival rate to the service rate of a single type of appliance, as a result of the inclusion of all \(M\) appliance types that are derived from the first equivalence. Since the TP policy of (19) considers a single threshold, we adjust our proposed scheduling policies to this constraint. Furthermore, the delay that power requests suffer in both OFF-TSP and ON-TSP is set to be equal to the deadline of power requests for TP (19). In Fig. 9 we present analytical results for the total number of requested p.u. for the 6 power consumption cases, under the
TP policy of (19) and the proposed OFF-TSP and ON-TSP. The results of Fig. 9 show that the outcome of TP application is the overestimation of the total power consumption, which is due to the fact that the TP policies consider a single and constant power request per residence; this assumption is doubtful in cases of multiple appliances types with ON-OFF functionalities.

5.3. Results of the total average cost

The previous evaluation examples are used in order to demonstrate the effect of the scheduling policies on the total average cost. To this end, we assume that the cost function for the baseline policy is \( C(\vec{j}) = 2 \cdot j_1^3 \). We also assume that the cost function for the two offline and the two online policies is identical and is defined as:

\[
C(\vec{j}) = \begin{cases} 
0.5 \cdot j_1^3, & \text{if } j_1 \leq P_1^{real} \\
1 \cdot j_1^3, & \text{if } P_1^{real} \leq j_1 \leq P_2^{real} \\
1.5 \cdot j_1^3, & \text{if } P_2^{real} \leq j_1 \leq P_3^{real} \\
2 \cdot j_1^3, & \text{if } P_3^{real} \leq j_1 \leq P \\
\end{cases}
\] (22)

Therefore, the cost function for the scheduling policies is set in such a way, so that the cost is gradually increased with the increase of the power consumption. The analytical results for the total average cost, over the 6 power consumption cases are depicted in Fig. 10. For the cases of OFF-PSP and OFF-TSP we consider that the third threshold \( P_3^{real}=500 \text{ p.u.} \) is applied. We observe that the total average cost for the cases D and E decreases for all the proposed policies, even though the total power consumption is increased. This is because the selected values of the arrival rates and operation times for the power requests in cases D and E are in total lower than the values that
correspond to the case C. Therefore, since the total average cost considers the entire set of the probabilities \( q(\vec{j}) = q(j_1, j_2) \), which are distributed in the range \( 0 \leq j_1 \leq P \) and \( 0 \leq j_2 \leq P^{fict} - j_1 \), the total average cost is lower for cases D and E, than the cost for case C. Moreover, we notice that the application of a scheduling policy significantly decreases the total average cost. Evidently, this reduction is highly affected by the amount of the power compression or the request delay. For example, by setting the compression parameters \( \omega^t_m \) at 10\% higher than the corresponding \( \omega^t_m \) parameters that are considered in Fig. 10, the total average cost is increased by 17.3\%.

6. Conclusion

We present and analyze offline and online scheduling policies in a smart grid environment. In both cases either power compression or request postponement are applied, in order to achieve reduction of the overall power consumption in a residential area. All scheduling policies assume that each residence is equipped with a specific number of appliances, with diverse power requests and operational times. Moreover, the proposed analysis considers types of appliances that alternate between ON and OFF states. For each scheduling policy we provide the analysis to determine the distribution of p.u. in use, and an appropriate dynamic cost function for the calculation of the total average cost. The accuracy of the proposed models is quite satisfactory, as it is verified by simulation. Furthermore, we demonstrate the advantages of the proposed models over the state of the art techniques. The proposed analysis can be used by power utilities, in order to make more cost effective scheduling decisions for the implementation of a smart and efficient
power grid.

Appendix A.

Appendix A.1. Single Threshold Case

For the derivation of Eq. (9) we firstly consider the case of one threshold $H$. Each residence is equipped with two groups of appliances: the first group consists of $M_1$ types of appliances that compress their power demands when the total power consumption exceeds the threshold $P_H$, while the second group consists of $M_2$ types of appliances with constant power demands, where $M_1 + M_2 = M$. Since appliances from the second group do not compress their power demands, the model of Eq. (3) can be applied; therefore the following local balance equation stands for a type-$m_2$ appliance of the second group:

$$f_{i,m_2}q(\vec{j} - R_{i,m_2}) = y_{i,m_2}(\vec{j})q(\vec{j}) 
\Rightarrow
f_{i,m_2}r_{i,m_2,s}q(\vec{j} - R_{i,m_2}) = y_{i,m_2}(\vec{j})r_{i,m_2,s}q(\vec{j})$$  \hspace{1cm} (A.1)

for $j_1 \leq P$ and $j_1 + j_2 \leq P_{\text{fict}}$

where $y_{i,m}(\vec{j})$ is the mean number of type-$m$ appliances that are active in the system, when the number of p.u. in use is $\vec{j} = (j_1, j_2)$, with $j_1 \leq P$ and $j_1 + j_2 \leq P_{\text{fict}}$. Due to the application of the power threshold $P_{\text{real}}$ (and the corresponding fictitious threshold $P_{\text{fict}}$), we define the following local balance equations for the case of a type-$m_1$ appliance that belongs to the first group:

$$f_{i,m_1}q(\vec{j} - R_{i,m_1}) = y_{i,m_1}(\vec{j})q(\vec{j}) 
\Rightarrow
f_{i,m_1}r_{i,m_1,s}q(\vec{j} - R_{i,m_1}) = y_{i,m_1}(\vec{j})r_{i,m_1,s}q(\vec{j})$$  \hspace{1cm} (A.2)

for $j_1 \leq P_H + r_{1,m_1}$ and $j_1 + j_2 \leq P_{\text{fict}} + r_{1,m_1}$
\[ f^{H,1}_{i,m} q(\vec{j} - R^{H,1}_{i,m}) = y^{H,1}_{i,m}(\vec{j}) q(\vec{j}) \]  
\[ f^{H,1}_{i,m} r^{i,m,1}_{i,m,1,s} q(\vec{j} - R^{H,1}_{i,m}) = y^{H,1}_{i,m}(\vec{j}) r^{i,m,1}_{i,m,1,s} q(\vec{j}) \]  
\[ \text{for } j_1 > P^{\text{real}}_{H} + r^{H,1}_{i,m,1} \text{ and } j_1 + j_2 > P^{\text{fict}}_{H} + r^{H,1}_{i,m,1} \]  

(A.3)

where \( y^{H,1}_{i,m} \) is the mean number of type-\( m \) appliances that are active in the system, when the number of p.u. in use is \( \vec{j} = (j_1, j_2) \), with \( j_1 > P^{\text{real}}_{H} + r^{H,1}_{i,m,1} \) and \( j_1 + j_2 > P^{\text{fict}}_{H} + r^{H,1}_{i,m,1} \). By considering that the simple case of \( M_1=1 \) and \( M_2=1 \), i.e. only one type of appliance per group exists in the system, Eq. (A.1) to Eq. (A.3) form the following system:

\[ f^{H,2}_{i,m} r^{i,m,2}_{i,m,2,s} q(\vec{j} - R^{H,2}_{i,m}) + f^{H,1}_{i,m} r^{H,1}_{i,m,1,s} q(\vec{j} - R^{H,1}_{i,m}) = \]
\[ (y^{H,2}_{i,m}(\vec{j}) r^{i,m,2}_{i,m,2,s} + y^{H,1}_{i,m}(\vec{j}) r^{i,m,1}_{i,m,1,s}) \cdot q(\vec{j}) \]  
\[ \text{if } 1 \leq j_1 \leq P^{\text{real}}_{H} + r^{H,1}_{i,m,1} \text{ and } j_1 + j_2 \leq P^{\text{fict}}_{H} + r^{H,1}_{i,m,1} \]  
\[ (A.4) \]

Since \( j_s = \sum_{i=1}^{2} \sum_{m=1}^{M} (y^{H,1}_{i,m}(\vec{j}) r^{i,m,1}_{i,m,1,s} + y^{H,1}_{i,m}(\vec{j}) r^{H,1}_{i,m,1,s}) \), i.e. the sum of all the products of the mean number of appliances to their corresponding power demands in all states is equal to the total number of p.u. in use, we introduce
the following approximations, in order for the sum of the right hand side of
Eq. (A.4), Eq. (A.5) and Eq. (A.6) to be equal to $j_s$: the mean number
$y_{i,m_1}^H(\vec{j})$ of appliances that require $r_{m_1}^H$ p.u. is negligible when $j_1 \leq P_{\text{real}}^H + r_{m_1}^H$. Also, the mean number $y_{i,m_1}(\vec{j})$ of appliances that require $r_{m_1}$ p.u. is
negligible when $P_{\text{real}}^H + r_{m_1} < j_1 \leq P$. Based on these two approximations
and by summing up side by side Eq. (A.4), Eq. (A.5) and Eq. (A.6) we
obtain:

$$f_{i,m_2} f_{i,m_2,s} q(\vec{j} - R_{i,m_2}) + f_{i,m_1} f_{i,m_1,s} \varphi_1(\vec{j}) q(\vec{j} - R_{i,m_1}) +$$
$$+ f_{i,m_1} f_{i,m_1,s} \gamma_1(\vec{j}) q(\vec{j} - R_{i,m_1}^H) = j_s q(\vec{j})$$

(A.7)

for $j_1 \leq P$ and $j_1 + j_2 \leq P_{\text{fict}}$. The functions $\varphi_1(\vec{j})$ and $\gamma_1(\vec{j})$ are used in
order to consider the aforementioned approximations, and are given by the
following expressions:

$$\varphi_1(\vec{j}) = \begin{cases} 1 & \text{if } \left\{ \begin{array}{l} (j_1 \leq P_{\text{real}}^H + r_{1,m_1} \cap j_1 + j_2 \leq P_{\text{fict}}^H \cap r_{1,m_1}^H > 0) \\ (j_1 + j_2 \leq P_{\text{fict}} \cap r_{1,m_1}^H = 0) \end{array} \right\} \\ 0 & \text{otherwise} \end{cases}$$

(A.8)

$$\gamma_1(\vec{j}) = \begin{cases} 1 & \text{if } \left\{ \begin{array}{l} (j_1 > P_{\text{real}}^H + r_{1,m_1}^H) \cap (j_1 + j_2 > P_{\text{fict}}^H + r_{1,m_1}^H) \cap \\ (r_{1,m_1}^H > 0) \end{array} \right\} \\ 0 & \text{otherwise} \end{cases}$$

(A.9)

By considering the general case of $M_1 + M_2 = M$ types of appliances, Eq.
(A.7) is finally written as:
\[
\sum_{i=1}^{2} \sum_{m=1}^{M} f_{i,m} r_{i,m,s} \varphi_{m}(\vec{j}) q(\vec{j} - R_{i,m}) + \\
+ \sum_{i=1}^{2} \sum_{m=1}^{M} f_{i,m}^{H} r_{i,m,s}^{H} \gamma_{m}(\vec{j}) q(\vec{j} - R_{i,m}^{H}) = j_{s} q(\vec{j})
\]  

(A.10)

where the functions \(\varphi_{m}(\vec{j})\) and \(\gamma_{m}(\vec{j})\) are defined by considering Eq. (A.8) and Eq. (A.9) and the fact that if a type-\(m\) appliance is not able to compress its demands \(r_{i,m}^{H} = 0\), then \(\varphi_{m}(\vec{j}) = 1\), and \(\gamma_{m}(\vec{j}) = 0\) in any system state.

Appendix A.2. Multi Threshold Case

For the case of multiple thresholds, a similar procedure is followed, as in the case of the single threshold case. More precisely, the local balance equations that are valid due to the presence of the thresholds are:

\[
f_{i,m} q(\vec{j} - R_{i,m}) = y_{i,m}(\vec{j}) q(\vec{j}) \iff \\
f_{i,m} r_{i,m,s} q(\vec{j} - R_{i,m}) = y_{i,m}(\vec{j}) r_{i,m,s} q(\vec{j})
\]

for \(j_{1} \leq P_{\text{real}}\) and \(j_{1} + j_{2} \leq P_{\text{fict}}\)  

\[
f_{i,m}^{h} q(\vec{j} - R_{i,m}^{h}) = y_{i,m}^{h}(\vec{j}) q(\vec{j}) \iff \\
f_{i,m}^{h} r_{i,m,s}^{h} q(\vec{j} - R_{i,m}^{h}) = y_{i,m}^{h}(\vec{j}) r_{i,m,s}^{h} q(\vec{j})
\]

(A.12)

for \(P_{h-1} + r_{1,m}^{h} \leq j_{1} < P_{h} + r_{1,m}^{h}\) and \(P_{h-1} + r_{1,m}^{h} \leq j_{1} + j_{2} \leq P_{h} + r_{1,m}^{h}\)

By following the same steps that were used in the derivation of the recursive formula of the single threshold case, and by considering all \(M\) types of appliances and all \(H\) thresholds, we derive Eq. (9). Note that the approximations that are considered in the multi threshold case are similar to the corresponding approximations that are considered in the single threshold case. Specifically, we assume that the mean number \(y_{i,m}(\vec{j})\) of appliances that require \(r_{m}\) p.u. is negligible when \(j_{1} > P_{1} + r_{m}\). Also, the mean number
$y_{i,m}^h(\vec{j})$ of appliances that require $r_{i,m}^h$ p.u., $h = 1, \ldots, H$, is negligible outside the region $P^{h-1}+r_{i,m}^{h-1} < j_1 \leq P^h+r_{i,m}^h$. These two approximations are expressed by Eq. (10) and Eq. (11), respectively. Please note that the two functions $\varphi_m(\vec{j})$ and $\gamma_m(\vec{j})$ consider that a type-$m$ appliance is not able to compress its demands ($r_{i,m}^h = 0$, $h = 1, \ldots, H$); therefore $\varphi_m(\vec{j}) = 1$, and $\gamma_m(\vec{j}) = 0$ in any system state.

References


[14] H. Goudarzi, S. Hatami, M. Pedram, Demand-side load scheduling in-


Fig. 1. Schematic diagram of a typical smart grid architecture.

Fig. 2. Schematic diagram of an appliance’s transition between ON and OFF states.
Fig. 3. Designation of power thresholds for the application of an offline scheduling policy.

Fig. 4. Analytical and simulation results for the baseline policy and the OFF-PSP.
Fig. 5. Analytical and simulation results for the baseline policy and the OFF-TSP.

Fig. 6. Comparison of analytical results for the baseline policy, the OFF-PSP and the OFF-TSP, under pre-defined thresholds.
Fig. 7. Analytical and simulation results for the baseline policy and the ON-PSP.

Fig. 8. Analytical and simulation results for the baseline policy and the ON-TSP.
Fig. 9. Comparison of the proposed OFF-TSP and OFF-TSP with the TP of [14].

Fig. 10. Analytical results for the total average cost, under the proposed scheduling policies.
Table 1: Types of appliances and corresponding parameters for the 6 power consumption cases.

<table>
<thead>
<tr>
<th>Power consumption case</th>
<th>Type of appliances parameters $m$: $\lambda_m$ (req./min.), $a_m$, $d_{1,m}$ (min.), $d_{2,m}$ (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3: (1.0, 0.9, 55, 5), 5: (1.6, 0.8, 30, 30), 7: (1.9, 0.0, 50, 0), 9: (1.9, 0.0, 40, 0), 10: (1.9, 0.0, 55, 0)</td>
</tr>
<tr>
<td>B</td>
<td>1: (1.3, 0.9, 55, 5), 3: (1.2, 0.9, 55, 5), 5: (1.6, 0.8, 30, 30), 6: (1.2, 0.6, 20, 40), 8: (1.7, 0.0, 55, 0)</td>
</tr>
<tr>
<td>C</td>
<td>1: (1.2, 0.9, 55, 5), 2: (1.4, 0.2, 45, 5), 4: (1.7, 0.5, 40, 20), 6: (1.5, 0.95, 30, 30), 7: (1.7, 0.0, 55, 0)</td>
</tr>
<tr>
<td>D</td>
<td>2: (1.0, 0.9, 55, 5), 3: (1.1, 0.2, 45, 15), 5: (1.5, 0.5, 50, 10), 6: (1.1, 0.95, 40, 20), 7: (1.4, 0.0, 55, 0)</td>
</tr>
<tr>
<td>E</td>
<td>1: (1.7, 0.9, 55, 5), 2: (1.7, 0.9, 50, 10), 3: (1.7, 0.9, 50, 10), 5: (1.6, 0.9, 55, 5), 8: (1.6, 0.0, 55, 0)</td>
</tr>
<tr>
<td>F</td>
<td>1: (1.6, 0.9, 50, 10), 2: (1.7, 0.9, 45, 15), 4: (1.6, 0.9, 50, 10), 6: (1.7, 0.9, 50, 10), 7: (1.9, 0.0, 55, 0)</td>
</tr>
</tbody>
</table>