A hybrid adaptive blind equalization for wireless communication

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Abstract-In this paper, a hybrid adaptive blind equalization method for wireless communication is proposed. For common blind equalizations, much attention has been paid on the additive noise (CMA, MCMA), and on the multi-path environment. The method in this paper combines those algorithms and has a good performance. Compared with other algorithms, this method can be used in severe multi-path environments for it has good decorrelation ability, at the same time makes full use of the characteristics of the communication signals, and in low SNR environment.

Keywords: Blind equalization, wireless communication, multi-path, CMA

I. INTRODUCTION

Wireless mobile communications face a difficult fading and multi-path environment that may cause severe signal distortions [1], [2]. While there are several circuit and algorithm components used to combat the channel impairments, one of the most important elements in the receiver is the adaptive equalizer. For wireless communication, the main concerns are convergence time and mean square error (MSE). The classical techniques either periodically send training sequences or use blind techniques. Adaptive equalization using training sequence lowers the bandwidth efficiency. But with blind equalization, no training is needed, and equalization is obtained only with the utilization of the received signal. A well-known blind equalization algorithm is the constant modulus algorithm (CMA) [1]. A main disadvantage of this algorithm is its convergence capability [1]. Some other algorithm based on blind decorrelation are widely used [3].

In this paper, a hybrid blind equalization method is proposed. It can be separated into two parts: a decorrelation part and a equalization part. The main mission of the first part is to correct the multi-channel influence in wireless communication channel. And the second part makes full use of the characteristics of communication signals (the distribution on the planisphere) to lower the additive noise.

II. THE COMMON ALGORITHM

In wireless mobile communication, multi-path channel is a difficult point. The channel is modeled by an FIR filter of N orders, whose coefficients are split as [1]

\[ \tilde{h} = h_0, h_1, \cdots, h_{N-1} \]  

(1)

it is the impose response of the channel. In a steady-state environment or environment with a small degree of nonstationarity, the coefficients are fixed in the convergence process.

The received signal can be expressed as [1]

\[ \tilde{x} = \tilde{s} * \tilde{h} + n \]  

(2)

In equation (2), \( \tilde{s} \) is the transmitted signal, \( n \) is the additive white Gauss noise.

Blind equalizations mitigate the channel distortion by extracting properties of the received signal. In CMA, the dispersion of the modulus of the equalizer output about a constant is minimized [4]

\[ J_c(w) = E \left\{ \frac{1}{4} \left[ |y|^2 - A \right]^2 \right\}, \quad A = E \left\{ |y|^2 \right\} \]  

(3)

The CMA algorithm updates the equalizer coefficient vector \( w \) by using the following equation:

\[ w_{k+1} = w_k - \mu \bar{x}_k \bar{x}_k^* \]  

(4)

where **"** denotes complex conjugation, \( \mu \) is the step size, and

\[ \bar{x}_k = x_k \left( |y|^2 - A \right) \]  

(5)

Under weak conditions, the CMA performance function is shown to characterize the ISI sufficiently well, whereas its stochastic minimization can be performed with no knowledge of the transmitted data. However, the algorithm’s performance after initial convergence is not satisfactory, due to the large residual variance of the error signal. For nonconstant modulus signal, the error variance could be intolerable [5].

Some algorithms are proposed to deal with this shortage (MCMA). The general form of the cost function for the MCMA is given by

\[ J_c(w) = E \left\{ \frac{1}{4} \left[ |y|^2 - A \right]^2 + \beta \left( g(x_w) + g(x_i) \right) \right\} \]  

(6)

where \( x_w \) and \( x_i \) are the real and imaginary parts of \( x_k \) respectively, \( g(x) \) is the constellation matched error function, and \( \beta \) is a weighting factor that trades between the amplitude and the constellation-matched errors.

The gradient recursion for the equalizer weight vector \( w \) can be in [5] formulated as

\[ w_{k+1} = w_k - \mu \bar{x}_k \nabla J_m(w) \bigg|_{w=w_k} \]  

(7)

where \( \mu \) is the step size.

One of the matched functions in [6] is as follows:

\[ g(x) = 1 - \sin^{2\pi} \left( \frac{x}{2d} \right) \]  

(8)

and the updating equation is

\[ w_{k+1} = w_k - \mu \phi_k \bar{x}_k \]  

(9)

\[ \phi_k = x_k \left( |y|^2 - A \right) \]  

(10)

\[ - \beta \frac{\pi}{2d} \left[ \sin \left( \frac{x_w}{d} \right) + j \sin \left( \frac{x_i}{d} \pi \right) \right] \]
where $2d$ is the minimum distance between symbols.

This algorithm is suitable for most of the communication signals. In correct signal point, $g(x)$ has the zero value. If the additive noise in existence, the value of $g(x)$ describes the information of noise.

The performance analysis of this algorithm is given in [5], using Taylor series expansion around the point $s_k$

$$g(x) = n \left( \frac{\pi}{2d} \right)^2 (z_k - s_k)^2$$

and

$$\phi_k = x \left( |x| - A \right) + \alpha (x - s_k) \quad \alpha = 2n \left( \frac{\pi}{2d} \right)^2 \beta$$

Those algorithms show good performance in the AWGN channel. Especially the MCMA in [5], it can work under a serious channel noise. Figure 1 and 2 show the simulation results.

Mutil-path channel is a great pain for wireless communication. Under this condition, most of the algorithms treat the inter-symbol interference (ISI) as additive noise, and cannot work very well.

For example, in equation (11), if ISI exist, we can find that

$$g(x) \neq n \left( \frac{\pi}{2d} \right)^2 (z_k - s_k)^2$$

for the high order terms cannot be ignored.

So under the mutil-path circumstance, blind equalization needs a method to cancel the mutil-path affects. Blind decorrelation is a useful way.

Assume that the source has the following characteristics:

$$E \{ s_i, s_j \} = 0 \quad \forall i \neq j$$

$$E \{ s_i^2 \} = 0 \quad \forall (i = 1, 2, \ldots)$$

If the equalizer converges, we can find that

$$y_k = w_k^T x_k = s_k$$

where $s_k$ is a estimate of signal $s_k$, and $y_k$ is the output of the equalizer.

So the equalized signal $y$ satisfy the characters:

$$E \{ y_i y_j \} = 0 \quad \forall i \neq j$$

$$E \{ y_i^2 \} = 0 \quad \forall (i = 1, 2, \ldots)$$

The following cost functions is permitted to use by theory [3]:

$$J_i (y, \omega_i) = \| \Lambda_i - \hat{R}_{y,y} \|_F$$

where $\Lambda_i = \text{diag} \{ \lambda_{i0}, \ldots, \lambda_{il} \}$, $\hat{R}_{y,y} = \langle y_i(k) y^T(k) \rangle$ is the cross correlation matrix with $y_i(k)= [y_i(k), y_i(k-1), \ldots, y_i(k-L)]^T$ and $y(k)= [y(k), y(k-1), \ldots, y(k-L)]^T$.

The updating equation of the blind decorrelation algorithm is [3]:

$$\Delta w_i (k) = \eta (k) \left[ \Lambda_i^{(i)} - \langle y_i(k) y^T(k) \rangle \right] w_i (k)$$

For the sensor signals corrupted by the additive Gaussian noise, instead of using cross-correlation matrix $\hat{R}_{y,y}$, we can use the following form [3]:

$$\Delta w_i (k) = \eta (k) \left[ \Lambda_i^{(i)} - \langle y_i(k) g^T(y(k)) \rangle \right] w_i (k)$$

where $g(y)=[g_0(y(k)), g_1(y(k-1)), \ldots, g_L(y(k-L))]^T$.

The learning rule can be formulated in a slightly modified form as:

$$\Delta w_i (k) = \eta (k) \left[ \Lambda_i^{(i)} - R_{y,y}^{(i)} \right] w_i (k)$$

where

$$\Lambda_i^{(i)} = (1 - \eta_0) \Lambda_i^{(i-1)} + \eta_0 \text{diag} \{ y_i(k) g^T(y(k)) \}$$

$$R_{y,y}^{(i)} = (1 - \eta_0) R_{y,y}^{(i-1)} + \eta_0 \langle y_i(k) g^T(y(k)) \rangle$$

To evaluate the performance of the proposed learning algorithms, the ISI ($M_{ISI}$) is defined as [3]
A hybrid blind equalization is proposed. It can be described as a two-order equalizer. It contains two parts: a decorrelation part and an equalization part.

The block diagram of the blind equalizer is as follows:

![Block diagram of the blind equalizer](image)

The received signals pass the first part, and then the ISI will be lower, and the equalizer takes the last ISI as additive noise.

The ISI of the output signal after decorrelation is not very severe (figure 4) [3], then the signal pass the equalizer can be expressed as equation (11).

Weight error of the equalizer is given by $\Delta w_k = w_k - w_{opt}$. Where $w_{opt}$ is the ideal equalizer vector. For a general adaptive algorithm:

$$E \left\{ \Delta w_{k+1} \right\} = E \left\{ \Delta w_k \right\} \quad \text{for } k \to \infty$$

In a steady state (as $k \to \infty$), we obtain

$$E \left\{ e^2(k) \right\} = E \left\{ \frac{1}{\|x\|_2^2} e^2(k) - \mu \|x\|_2^2 \right\}$$

where $e(k) = \Delta w_k^H x_k$.

By dropping $k$ for simplicity, the above equation can be expanded as

$$E \left\{ e^2 \right\} = E \left\{ e^2 \right\} - \frac{\mu E \left\{ e^2 + e^T \varphi \right\} + \mu^2 E \left\{ x \right\} \| \varphi \|_2^2}{T_1}$$

This implies that $T1$ and $T2$ are identical. If ISI is not severe, equation (11) is correct, then replace $x$ by $s + e$, (12) can be expressed as:

$$\varphi_k = \left( \left( s + e \right) - A \right) \left( s + e \right) + \alpha e$$

The terms in (25) can be computed.

$$T1 = 2 \mu E \left\{ 2 \left( s + A \right) \| e \|_2^2 \right\} + \mu E \left\{ s \| e \|_2^2 \right\} + 3 \mu E \left\{ s^T e^2 \right\} - A \mu E \left\{ s e^T + s^T e \right\}$$

$$\approx 2 \mu E \left\{ 2 \right\} \| e \|_2^2$$

$$T2 = \mu^2 E \left\{ 9 \left( s + A \right) \| e \|_2^2 \right\} + \mu^2 E \left\{ 9 \left( s + A \right) \| e \|_2^2 \right\}$$

$$\approx \mu^2 E \left\{ 9 \right\} \| e \|_2^2$$

Under the same conditions, common CMA and MCMA will not converge.

### III. HYBRID METHOD
Since $T_1 = T_2$, then

$$E\{|e_2|^2\} = \frac{E\{|h|^4 - 2A|h|^2 + A^2\}}{2E\{|h|^4 - A + \alpha|^2\} - E\{|h|^4 + 4(\alpha - 2A)|h|^2 + (\alpha - A)^2\}}$$

In fact, for the ISI still exist, $E\{|e_2|^2\}$ is much greater than in [5], and the final $E\{|e_2|^2\}$ will not reach to a small value. But it is still much lower than that of the common algorithms.

The following figures show the result of the two-order equalization comparing with those of common algorithms.

Figure 6 is the planisphere of a QPSK signal pass through the two-order equalizer, it is much resolube.

Figure 7 is the comparison of the MSE of the common algorithm and the two order equalization.

Figure 6 QPSK signal passing through two-order equalizer

![Figure 6](image)

Figure 7 MSE compare. The common equalizer uses the CMA algorithm; the MSE of two order equalizer is less than the CMA algorithm.

![Figure 7](image)

IV. CONCLUSION

The two order equalization has a better performance than the common equalization algorithm. The main shortage of this method is that the system will be more complex. It is suitable for the steady-state environment or environment with a small degree of nonstationarity. For the blind equalization, the convergence rate is too slow. It needs to know the statistical parameters, which this needs a long time to be gained.

REFERENCES


