ML Time Synchronization Algorithm Joint with Channel Estimation for MIMO-OFDM Systems in Frequency Selective Fading Channel

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Abstract: This paper addresses Maximum-Likelihood (ML) time synchronization algorithm joint with channel estimation for MIMO-OFDM systems in frequency selective fading channels. For this purpose a SISO joint Maximum likelihood (ML) synchronization algorithm with channel estimation is extended and generalised to MIMO systems. Timing failure probability is presented as a criterion to illustrate the robustness of the algorithm. The performance of the proposed synchronization approach, in terms of timing failure probability, was compared with SISO systems. Based on simulation results, the performance of the proposed algorithm is quite satisfying.

1. INTRODUCTION

The technique of Multiple Input Multiple Output (MIMO) combined with Orthogonal Frequency Division Multiplexing (OFDM) in communication systems for multimedia applications have gained considerable interest in recent years. Although OFDM is well known for its ability to combat inter-symbol interference (ISI) introduced by multipath channel, incorrect positioning of the FFT window within an OFDM symbol reintroduces ISI during data demodulation, causing serious performance degradation [4]. Time synchronization is therefore one of the important tasks performed at the receiver.

A number of methods for OFDM time synchronization have been proposed in the literature [1]–[14]. A lot of them are based on preamble repetition scheme introduced by Moose [2]. He described a technique to estimate frequency offset by using a repeated training sequence and derived maximum likelihood estimation metrics. Then, this idea was employed for time synchronization by Schmidle [3], Speth [4] and Keller and Hanzo[5]. The principle of exploiting a double training sequence preceded by a cyclic prefix (CP) for frame synchronization was originally suggested for single-carrier transmission in [6]. Weinfurtner in [7] introduced and compared several metrics to detect repeated preamble in the received sequence for frame synchronization. A simple MIMO extension of Schmidl’s algorithm was proposed in [8], and another MIMO extension of repeated preamble algorithm was utilized by Zelst [9]. But all of these algorithms have one disadvantage in common. These algorithms, as a matter of fact, use auto correlation criterion that result in a plateau during cyclic prefix [10], so they are not able to determine the precise packet arrival time and we have to perform symbol timing after these algorithms. Some authors propose the use of periodic structure of cyclic prefix in OFDM symbols [12], but these kinds of algorithms can not be extended over MIMO systems easily.

On the other hand, there are some time synchronization techniques that are specifically designed for IEEE 802.11a WLANs standard [13],[14]. Although some of these algorithms like [14], work well but they have benefited from the structure of this standard so they can not be applied to a generic OFDM system and they have been designed for SISO systems.

Simple correlator can be easily implemented at the receiver, but its performance is poor in dispersive and unknown channels [15], indicating that more sophisticated synchronization algorithms are required. This paper is our effort to establish a more complex time synchronization algorithm in MIMO–OFDM systems with unknown frequency selective channel which is elaborated from the ideas of [11] and [14]. The former is the ML time synchronization algorithm and the latter is the ML time synchronization algorithm for IEEE 802.11a WLANs both for SISO systems. In fact we extend and generalize the method of these algorithms, to develop a maximum-likelihood (ML) symbol synchronizer for a general MIMO–OFDM system on frequency selective channels. To the best of our knowledge, there is no published paper that proposes ML time synchronization/ channel estimation for MIMO–OFDM systems.

The rest of this paper is as follows. In section II, we describe our signal model for which the ML algorithm is tailored. Section III introduces our ML time synchronization and channel estimation algorithm. Section IV presents the simulation results and finally conclusions are drawn in section V.

2. SIGNAL MODEL

Consider a MIMO-OFDM communication system with $N_t$ transmit and $N_r$ receive antennas. At each receiving antenna, a superposition of faded signals from all the transmit antennas plus noise is received. Let the baseband equivalent signal of preamble at $n^{th}$ transmitter be $s_d(n)$, and $h_m(n)$ be the equivalent channel between $n^{th}$ transmit antenna and $m^{th}$ receive antenna. The
received signal $r_{sd}(t)$ at $m$th receive antenna is sampled at $t=kt_s+\varepsilon_t T_s$, where $T_s$ is the sampling time, and $\varepsilon_t \in [0,1)$ is the unknown time offset induced by the combination of the channel first path delay and the sampling phase offset. According to [16], if the equivalent channel bandwidth $(B_s)$ satisfies $B_s<1/T_s-B_s$ ($B_s$ is the bandwidth of $s(t)$), then by the equivalence of digital and analog filtering for band-limited signals, the sample received signal can be expressed as:

$$r_{sd}(t) = \sum_{k=1}^{N} s_k(t) h_{mn}(k) + w_{mn,k} + w_{sd}(t)$$

where $r_{sd}(t) \leftrightarrow r_{sd}(t) = h_{mn}(t) w_{mn,k} + w_{sd}(t)$ and $w_{sd}(t)$ is the stationary additive Gaussian noise at $m$th receive antenna which is independent of the other antennas.

The fading multi-path channel is considered to be quasi static. Supposing the channel length $L$, we have $N\times N$, paths, each of which can be modeled by an equivalent FIR complex filter of order $L$ with $h_{mn}(l)=f_{mn}(l)$ as the taps with $l=0,1,\ldots, L-1$. These taps are assumed to be independent zero mean complex Gaussian random variables with variance 1/2P(l) per dimension. The ensemble $P(l)$ with $l=0,1,\ldots, L-1$ is called the power delay profile (PDP) of the channel and its total power is assumed to be normalized to $\sigma^2=1$, which is the average channel attenuation. Therefore (1) can be written as:

$$r_{mn} = \sum_{k=1}^{N} s_k(t) h_{mn}(1) + w_{mn,k}$$

Let $\mathbf{r}_{mn}$ be a received-signal vector of size $N$, sampled from time $k$ to $N+k-1$ at $m$th antenna:

$$\mathbf{r}_{mn} = \begin{bmatrix} r_{m,k} & r_{m,k+1} & \cdots & r_{m,k+N-1} \end{bmatrix}^T \quad (N \times 1)$$

Let the training sequence of $n$th transmit antenna be $\mathbf{C}_{n} = \begin{bmatrix} C_{n,0} & C_{n,1} & \cdots & C_{n,N_p-1} \end{bmatrix}$ where $N_p$ is the length of training sequence. If the length of the cyclic prefix for the OFDM symbol is $N_{CP}$, we have $\mathbf{C}_{n} = \begin{bmatrix} C_{n,N_p} \cdots & C_{n,N_p-1} \end{bmatrix}$ as the cyclic prefix of training sequence. Supposing $-N_{CP} \leq k \leq N_{P} - N$, we have:

$$\mathbf{r}_{mn} = \sum_{n=1}^{N} \mathbf{C}_{n} h_{nm} + \mathbf{w}_{mn,k}$$

where:

$$\mathbf{C}_{n,k} = \begin{bmatrix} C_{n,\text{mod}(0,N_p)} & C_{n,\text{mod}(1,N_p)} & \cdots & C_{n,\text{mod}(L-1,N_p)} \\ C_{n,\text{mod}(1,N_p)} & C_{n,\text{mod}(2,N_p)} & \cdots & C_{n,\text{mod}(L-2,N_p)} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n,\text{mod}(L-1,N_p)} & C_{n,\text{mod}(L,N_p)} & \cdots & C_{n,\text{mod}(2L-N_p,N_p)} \end{bmatrix}$$

$$\mathbf{h}_{mn} = \begin{bmatrix} h_{mn}(0) & h_{mn}(1) & \cdots & h_{mn}(L-1) \end{bmatrix}^T$$

and $\mathbf{w}_{mn,k}$ is a column vector containing the noise samples at the $m$th receive antenna with covariance matrix $\mathbf{\sigma}^2 \mathbf{I}_N$ ($\mathbf{I}_N$ is the $N \times N$, identity matrix).

Stacking the received vectors from all the $N_r$ receive antennas and using Kronecker product, we have:

$$\mathbf{r}_c = (I_{N_r} \otimes \mathbf{C}_z) \mathbf{h}_b + \mathbf{w}_s$$

where:

$$\mathbf{r}_c = \begin{bmatrix} r_{1a}^T & r_{2a}^T & \cdots & r_{N_{r}a}^T \end{bmatrix}^T \quad (N_r \times 1)$$

$$\mathbf{C}_k = \begin{bmatrix} C_{1,k} & C_{2,k} & \cdots & C_{N_r,k} \end{bmatrix} \quad (N_r \times N_r)$$

$$\mathbf{h}_b = \begin{bmatrix} \mathbf{h}_{1}^T & \mathbf{h}_{2}^T & \cdots & \mathbf{h}_{N_r}^T \end{bmatrix} \quad (L \times N_r)$$

$$\mathbf{w}_s = \begin{bmatrix} \mathbf{w}_{1s}^T & \mathbf{w}_{2s}^T & \cdots & \mathbf{w}_{N_{r}s}^T \end{bmatrix} \quad (L \times N_r)$$

3. ML SYNCHRONIZER

Frame synchronization is to identify the preamble in order to detect packet arrival. This preamble detection algorithm can also be used as a timing algorithm, since it inherently provides an estimate of the starting point of the packet.

There are several methods proposed in the literature to perform frame synchronization. In this section we are to propose a new algorithm for this purpose. The advantages of our algorithm, as we will see later, are its accuracy and the fact that there is no constraint over the structure of preamble.

The idea of calculation of complex correlation of two preambles, proposed in literature, is a good way to detect packet arrival. This method calculates the correlation of received signal with itself with a delay of $N$ samples. The drawback of this method is that the correlation function does not produce a sharp peak at the arrival of the data packet. In fact, during the cyclic prefix period, the correlation function remains constant and there is no abrupt falling edge outside this period. This edge can not be localized precisely in a cheap signal quality. This phenomenon, inherent to the structure of training sequence, makes it difficult to find the fine timing. On the other side the performance of cross correlation method, which produces a sharp peak, is poor in dispersive and unknown channels [15]. We propose, therefore, a new robust solution to find the exact time of packet arrival in unknown dispersive channels.

We have seen in equation (7) that the received vector can be written as a function of the preamble, channel matrix and the noise. Now suppose we have a sliding window with length $N$ to observe the received vector. $\mathbf{r}_k$ is the observed received vector starting from position $k$ to $k+N$ (Figure 1).

We supposed that the beginning of the preamble, as shown in figure 1, is chosen as the time reference, i.e. $k=0$. The objective, then, is to find the instant $k=0$. For this purpose we have to maximize the probability of receiving $\mathbf{r}_k$ on
condition of sending a known vector, e.g. the preamble. By using equation (7) we can calculate this probability as follows:

\[
p(\mathbf{r}_k | [k, \mathbf{h}]) = \frac{1}{(2\pi\sigma_v^2)^N} \exp\left(-\frac{\|\mathbf{r}_k - (I_{N_r} \otimes C_0)\mathbf{h}\|^2}{2\sigma_v^2}\right)
\]

where \(C_0\) is the training sequence matrix and by using (9) can be written as:

\[
C_0 = \begin{bmatrix}
    C_{1,0} & C_{2,0} & \ldots & C_{N_r,0}
\end{bmatrix}_{N \times N_r}
\]

By varying \(k\) and maximizing the probability of equation (12) the arrival instant of the beginning of the packet can be estimated. To estimate the arrival instant, instead of maximizing the probability we can equivalently minimize following metric:

\[
J(\mathbf{r}_k | [k, \mathbf{h}]) = (\mathbf{r}_k - (I_{N_r} \otimes C_0)\mathbf{h})^H (\mathbf{r}_k - (I_{N_r} \otimes C_0)\mathbf{h})
\]

Setting the partial derivative of \(J(\mathbf{r}_k | [k, \mathbf{h}])\) with respect to \(\mathbf{h}\) to zero, the ML estimate for \(\mathbf{h}\) (when \(k\) is fixed) is obtained as [17]:

\[
\hat{\mathbf{h}} = (I_{N_r} \otimes C_0)^H (I_{N_r} \otimes C_0)^{-1} (I_{N_r} \otimes C_0)^H \mathbf{r}_k
\]

In order that this equation leads to unique solution, the number of observation points must be more than the number of unknown channel coefficients i.e.: \(N \times N_r > L \times N_r \times N_r\).

Substituting equation (15) into equation (14), after some straightforward manipulations and dropping the irrelevant terms, the packet arrival time is estimated by maximizing the following likelihood function:

\[
\Psi(k) = \mathbf{r}_k^H (I_{N_r} \otimes C_0)^H (I_{N_r} \otimes C_0)^{-1} (I_{N_r} \otimes C_0)^H \mathbf{r}_k
\]

Using the well-known properties of the Kronecker product \((A \otimes B)^H = A^H \otimes B^H\), \((A \otimes B)^H = A^H \otimes B^H\), and \((A \otimes B)(C \otimes D) = (AC) \otimes (BD)\), we have:

\[
(I_{N_r} \otimes C_0)^H (I_{N_r} \otimes C_0)^{-1} (I_{N_r} \otimes C_0)^H = I_{N_r} \otimes C_0 (C_0^H C_0)^{-1} C_0^H
\]

Substituting this result into equation (16) the likelihood function is:

\[
\Psi(k) = \mathbf{r}_k^H (I_{N_r} \otimes C_0 (C_0^H C_0)^{-1} C_0^H) \mathbf{r}_k = \sum_{i=1}^{N_r} \mathbf{r}_k^H C_0 (C_0^H C_0)^{-1} C_0^H \mathbf{r}_k
\]

The arrival packet instant \((k=0)\) from the received signal vector \(\mathbf{r}_k\) can be calculated as follows:

\[
\hat{k} = \arg \max_k \Psi(k)
\]

We make following remarks:

- In our algorithm there is no constraint over training sequence structure. It means that we can use one preamble or two consecutive repeated preambles like [2]-[9].
- The orthogonality between the preambles of different antennas is not required.
- By calculating arrival instant, the channel can be estimated by using (15).

4. SIMULATION RESULTS

As the performance of our ML time synchronization algorithm we can calculate the probability of finding the correct sampling instant. This lock-in probability can be represented mathematically as follows:

\[
P_{\text{lock-in}} = \Pr\{\hat{k} = 0\}
\]

Since in a Rayleigh multipath fading channel, the channel may contain some small taps at the beginning, so the starting position of the channel is not clear. So instead of lock-in probability, we introduce another criterion to reveal the probability that our time synchronizer calculated instant is far from the exact instant of packet arrival. Timing failure \((t_f)\) probability is introduced similar to [7] as an indicator to illustrate the robustness of our algorithm. \(P_{t_f}^{(2)}\) is the probability that the frame synchronizer misses an interval of \(2p+1\) samples centered at \(k_0\). This probability is expressed as follows:

\[
P_{t_f}^{(2)} = \Pr\{|\hat{k} - p| > p\}
\]

For the simulation setup we use the same parameters as [7], that is, the length of OFDM symbol is 64 and there is a cyclic prefix for each OFDM symbol with length 8. Exponentially decaying channel power delay profile of length 8 is used, decaying 3 dB per tap in positive time direction. The channel is fixed during transmission of one packet and independent of that of another packet. In contrast with [7], we use a \(4 \times 4\) MIMO system and it is assumed that the average total TX power \(P\) is distributed among the TX antennas such that \(\sigma_u^2 = P/N_t\). The SNR per receive antenna is \(P/\sigma_u^2 = N_t\). As preamble, we use a single sequence with the length of an OFDM symbol and a cyclic prefix of length 8 while in [7] two preambles were
used. The preamble is generated randomly for each transmit antenna. Our sampling vector length, $N$, is 64 and each point of result is obtained by averaging over $10^5$ Monte-Carlo runs.

Figure 2 presents our simulation results for a $4 \times 4$ MIMO together with the reported results in [7]. The probability to miss a $\pm 5$ and a $\pm 15$ interval is plotted in the figure. It is clear from the figures that the performance of the proposed algorithm is even better than the SISO case. Simulation results of lock-in probabilities are given in figure 3 for various SNR.

5. CONCLUSION

The problem of time synchronization and channel identification for MIMO-OFDM systems in unknown frequency selective is addressed in this paper. For this purpose a SISO joint Maximum likelihood (ML) synchronization algorithm with channel estimation is extended and generalised to MIMO systems over frequency selective fading channels. The principles of proposed algorithm can be applied to both frame synchronization and OFDM symbol timing. No constraint over preamble structure is required in presented algorithm. The performance, in terms of timing failure probability, of the proposed synchronization approach was compared with SISO systems. Simulation results show that this MIMO synchronization algorithm outperforms the equivalent one developed for SISO OFDM systems.

REFERENCES


