SEGMENTATION USING ADAPTIVE THRESHOLDING OF THE IMAGE HISTOGRAM
ACCORDING TO THE INCREMENTAL RATES OF THE SEGMENT LIKELIHOOD
FUNCTIONS

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ABSTRACT

A novel algorithm for image segmentation based on adaptive thresholding of the global histogram of an image is proposed and applied to medical images from the medical database of the Second Department of Surgery of the University Hospital of Alexandroupolis, Greece. The threshold values are specified through an adaptive process that determines the optimum number of histogram regions and, at the same time, attempts to minimize the optimal likelihood value obtained from the specific partition of the histogram. The main peaks of the histogram are selected as seeds for the initial partition of the histogram. These seeds are subsequently grown by varying their upper and lower boundaries according to the incremental changes of likelihood values corresponding to the different intervals of the image histogram. The proposed method provides an alternative way of selecting the dominant peaks of the image histogram according to predefined constraints.

1. INTRODUCTION

Image segmentation consists of determining \( K \) disjoint segments of an image, denoted as \( I \), that are compact, feature smooth boundaries and are homogeneous regarding the statistics of the pixel values within each region,

\[
P(I) = \{ R_1, R_2, R_3, \ldots, R_K \},
\]

where \( R_i \cap R_j = \emptyset \) with \( i, j \subseteq [1, K] \) and \( i \neq j \).

Image segmentation is an essential processing step inherent in a variety of algorithms that are intended for image enhancement (in the context of acquisition of medical imagery, outdoor and night vision imaging systems etc), for automatic pattern recognition (as implemented in radar and sonar systems, in medical diagnostic systems or in the context of automatic recognition of machine printed or handwritten texts), for shape recognition (as implemented in the context of robot vision and low-level vision) for coding of video sequences and still images (MPEG-4/H.264) and for a variety of other image processing tasks. Image segmentation methods may be classified into the following categories:

- Histogram thresholding using two or more thresholds based on the peaks and the valleys of the global histogram of an image [1]. Histogram thresholding may be crisp or fuzzy [2], [3].
- Local filtering approaches such as the Canny edge detector [4] and similar techniques.
- Region-growing and merging techniques based on pixel classification in some feature space [5], [6].
- Deformable model region growing [7].
- Global optimization approaches based on energy functionals [8] and/or mixture models of individual component densities (usually Gaussians). These approaches employ such techniques as Bayesian/Maximum a-posteriori criteria [9], the Expectation Maximization (EM) Algorithm [10], propagating fronts/level set segmentation [11], [12] and Minimum Description Length (MDL) criteria.
- Morphological methods like watersheds, morphological image analysis [13], [14] and hybrid morphological-statistical techniques [15].
- Fuzzy/rough set methods like fuzzy clustering and others [2], [3].
- Methods based on Artificial Neural Networks (ANNs) like unsupervised learning and evolutionary/genetic algorithms [16].
- Hybrid methods that attempt to unify several of the above approaches.

Particular segmentation algorithms are, generally, not applicable to all images. Practice shows that a specific method may yield segmentation results of varying quality when applied to images with different characteristics. This implies that different algorithms are not equally suitable for a specific application.
Histogram thresholding may be either crisp or fuzzy. A set of \( K \)-1 thresholds, denoted as \( \{T_1, T_2, \ldots, T_{K-1}\} \), is defined in order to segment an image into \( K \) segments, denoted as \( \{R_1, R_2, \ldots, R_K\} \), where
\[
I(m, n) \in R_k : k = 1, 2, \ldots, K \Rightarrow I(m, n) \in [T_{k-1}, T_k). \quad (2)
\]
\( [T_0, T_K) \) is the dynamic range of the pixel values of the image. Histogram thresholding assumes that pixels of the image featuring comparable gray-level values are well confined within compact regions of the image. Since this is not always the case, further processing of the segmented image via thresholding may be required.

Determining the appropriate thresholds that yield an efficient segmentation is a key issue. The global image histogram may be considered as a mixture of individual component densities, usually Gaussians. The Expectation Maximization (EM) Algorithm ([10]) is usually employed among other available techniques in order to estimate the free parameters of the individual component densities. The dominant peaks of the image histogram dictate the number of the individual component densities that comprise the image histogram should the mixture model be adopted. The thresholds are placed at gray-level values that correspond to deep valleys of the image histogram. The relative heights of the peaks of the image histogram are usually employed in order to distinguish between dominant and minor peaks whereas fuzzy membership functions, fuzzy methods and various fuzzy measures are employed in order to determine the partition of the image histogram into multiple threshold intervals [17].

Thresholds are determined by minimizing a measure of fuzziness, like the Shannon’s Entropy or the Index of Fuzziness, over some parameterization of the fuzzy membership functions that depends upon a set of thresholds. Let \( I(m, n) \) denote the gray-level value of a pixel of an image where \( m=1 \ldots M \) and \( n=1 \ldots N \). Shannon’s entropy [18] is given as,
\[
E = \frac{1}{MN} \ln K \sum_{k=1}^{K} \sum_{(m,n)=1}^{M,N} S_k(\mu_k(I(m,n))) \cdot \quad (3)
\]
where \( S_k \) is defined for each fuzzified segment of the image at \( (m,n) \) according to the following relationship,
\[
S_k(\mu_k(I(m,n))) = -\mu_k(I(m,n)) \ln \mu_k(I(m,n)) \cdot \quad (4)
\]
Shannon’s entropy assumes values from the interval \([0, 1]\). It has a minimum value \( 0 \), if \( \mu_k(m,n)=0 \) or 1 for all \( (m,n) \) and \( k \), and a maximum value 1, if \( \mu_k(m,n) = \frac{1}{K} \) for all \( (m,n) \) and \( k \). As an alternative to Shannon’s entropy,
Yager proposed in [19] a measure of fuzziness (the so-called Yager’s Fuzzy Index) that depends upon the relationship between a fuzzy set and its complement.

Several parameterizations are possible regarding the fuzzy membership functions provided that \( \sum_{k=1}^{K} \mu_k(I(m,n)) = 1 \ \forall \ (m,n) \). The standard \( S \)-functions are usually employed in order to derive the membership functions and, consequently, the optimal thresholds in conjunction with an optimization algorithm that minimizes either Shannon’s Entropy and Yager’s Fuzzy Index. Minimization is carried out over the dynamic range of the pixel values. The standard \( S \)-functions, which are used to derive the membership function of image segment \( k \)
\[
\mu_k(I(m,n)) = S_k(I(m,n); c_{k-1}, b_{k-1}) - S_k(I(m,n); c_k, b_k),
\]
are parameterized as,
\[
S_k(l(m,n), c_k, b_k) = \begin{cases} 0, & l(m,n) \leq c_k - b_k/2 \\ \frac{1}{2} \left[ (l(m,n) - c_k)/b_k + 1/2 \right]^2, & c_k - b_k/2 < l(m,n) \leq c_k \\ -\frac{1}{2} \left[ (l(m,n) - c_k)/b_k + 1/2 \right]^2, & c_k < l(m,n) \leq c_k + b_k/2 \\ 1, & c_k + b_k/2 < l(m,n) \end{cases}
\]
where \( c_k \) is the cross-over point of the \( S \)-function, i.e. the threshold between image segments \( R_{k-1} \) and \( R_k \), and \( b_k \) is its bandwidth (steepness or fuzzy dynamical range between the two segments). Obviously, the shape of the \( S \)-function is determined by these two parameters.

3. ADAPTIVE THRESHOLDING OF THE IMAGE HISTOGRAM ACCORDING TO THE GROWING RATE OF REGIONAL LIKELIHOOD FUNCTIONS

3.1. Definition of the partial likelihood functions
The proposed algorithm defines a partial likelihood function associated with each segment of the image or, equivalently, with each adaptively growing interval of the image histogram. The partial likelihood function for the \( k \)-segment reads,
\[
I_i(\theta_i) = \int_{R_k} \log p_i(I(m,n) | \theta_i) dR_k = \int_{R_k} \log p_i(g | \theta_i) dI_i
\]
where vector \( \theta_i \) holds the mean (denoted as \( \mu_i \)) and the standard deviation (denoted as \( \sigma_i \)) pertaining to \( R_k \) and \( T_k \) are the thresholds defining segment \( R_k \) according to Eq. 2 and \( dI_i \) is the number of pixels of the image with a gray-level value of \( g \).

The proposed algorithm is initiated from the dominant peaks of the image histogram and increases adaptively the interval that defines segment \( R_k \). The lower and the upper limits, which define interval \( \Delta R_k \) corresponding to segment \( R_k \) at iteration \( t \), are denoted as
The values of the partial likelihood functions at $t$ are minimized over the parameters of the individual component densities, i.e. $\theta_k=[\mu_k, \sigma_k]$ for $k=1,2,..K$. The proposed algorithm assumes that the slopes of the optimized, i.e. minimal, partial likelihoods, which are denoted as $l'_k(t)$, are increased monotonically.

The increase rate of the partial likelihoods is the same for all segments. A penalty parameter $\kappa$ is assigned to each segment in order to allow for segment (interval) merging.

### 3.2. The proposed algorithm for adaptive thresholding of the image histogram

The proposed algorithm is detailed in this section as described in the following steps. A list of the values of the partial likelihood functions corresponding to the segments of the image, denoted as $L_t = \{l_{1,t}, l_{2,t}, l_{3,t}, \ldots, l_{K,t}\}$, is formed at $t$. The partial likelihood functions are optimized over their parameters.

It turns out that the minimum values of the partial likelihoods depend upon the optimum values of the standard deviations for the case of Gaussian distributions, i.e.

$$l'_{k,t} = l_{k,t}(0) = N_{k,t} \times \left[ \log(\sqrt{2\pi}\sigma_{k,t}) + \frac{1}{2} \right]$$  \hspace{1cm} (7)

where $N_{k,t} = \int_{l_{k,t}}^{l_{k+1,t}} d\mu_{k,t}$. This yields a corresponding list $L'_t = \{\sigma_{1,t}(\lambda_{1,t}), \sigma_{2,t}(\lambda_{2,t}), \ldots, \sigma_{K,t}(\lambda_{K,t})\}$ where $\lambda_{k,t} = \frac{\Delta l'_{k,t}}{\Delta N_{k,t}} = \frac{l'_{k+1,t} - l'_{k,t}}{N_{k+1,t} - N_{k,t}} = \lambda$. The proposed algorithm assumes that the slopes of all segments are equal at $t$ and that the slope values increase monotonically over time. The corresponding list of the gray-level intervals at $t$ is defined as $L_t = \{\Delta g_{1,t}, \Delta g_{2,t}, \Delta g_{3,t}, \ldots, \Delta g_{K,t}\}$. The steps are detailed as follows:

**0:** Initialize the algorithm selecting the gray-level values of the histogram peaks as seeds, i.e. $L_0^0 = \{g(\text{Peak}_1), g(\text{Peak}_2), \ldots, g(\text{Peak}_K)\}$.

**1:** Find the minimum values of the partial likelihoods for $L_t^0$ and determine the list $L_t^0 = \{\sigma_{1,t}(\lambda_{1,t}), \sigma_{2,t}(\lambda_{2,t}), \ldots, \sigma_{K,t}(\lambda_{K,t})\}$.

**2:** Increase the histogram intervals $\Delta g_t$, where $k=1,2,..K(t)$, in such a way that $\lambda_{k,t} = \lambda_i \forall k$. The slope $\lambda_i$ has to increase monotonically as the algorithm proceeds.

**3:** If some upper boundary $u_{p,t}(t)$ coincides with lower boundary $l_{low}(t)$, set threshold $T_k$ in the set of thresholds. Adjust the gray-level values of already set thresholds in the set in order to minimize the overall likelihood.

**4:** Merge $\Delta g_{k,t}$ with $\Delta g_{k+1,t}$ (i.e. $\Delta g_{k,t} = \Delta g_{k,t} \cup \Delta g_{k+1,t}$ ) if $N_{k,t} \log\left(\frac{\sigma_{k,t}}{\sigma_{k+1,t}}\right) + N_{k+1,t} \log\left(\frac{\sigma_{k+1,t}}{\sigma_{k,t}}\right) < 2\kappa$. Determine the new s.t.d., denoted as $\sigma_{m,t}$, which corresponds to $\Delta g_{m,t}$. Reduce $K(t)$ by one.

**5:** Stop if the intervals corresponding to the segments of the image cover the entire dynamic range of the pixel values of the image, i.e. if $\bigcup_{k=1}^{K} \Delta g_{k,t} = [T_0, T_K]$, otherwise set $t = t + 1$ and go to Step-1.

### 4. EXPERIMENTAL RESULTS

The image shown in Fig. 1, is an X-ray radiograph from a medical database that has been developed in the Second Department of Surgery of the University Hospital of Alexandroupolis, Greece [20]. The patient is suspected to have perforation of a gastroduodenal ulcer. Plain radiographs are taken with the patient in the upright position in such a case. The air escapes from the perforation freely into the peritoneal cavity and collects under the diaphragm. It takes typically the form of a semilunar dark area limited between the right hemidiaphragm and the upper border of the liver or between the left hemidiaphragm and the rim of the gastric fundus. Intraperitoneal perforation of a gastroduodenal ulcer is not clearly presented in this image. Approximately 20% of patients with perforated gastroduodenal ulcer do not show free intraperitoneal air presumably because the adjacent omentum or other viscera seal the perforation before a significant amount of peritoneal air escapes. Technically poor radiographs due to miscalculated exposure conditions may also result in false negative images for such severely ill patients. The absence of free air in the abdomen may lead to a misdiagnosis especially in atypical cases. Since all patients with perforated gastroduodenal ulcer should undergo an emergency operation, it is extremely important to make the correct diagnosis. Therefore, false negative radiographs in which the free intraabdominal air is not visible due to technical reasons should be excluded. In our study, after acquisition of the plain radiographs, the films are scanned with a Heidelberg Lynotype CPS Saphir/Opal scanner in multiple resolutions, in order to achieve the best image according to the directions of the surgeons. It turns out that further processing of the
images by the application of the proposed algorithm facilitates the diagnosis.

The histogram of the image is given in Fig. 2.a. The gray-level values corresponding to the three salient peaks of the histogram at 2.0, 169.3 and 225.1, which are marked with the dashed lines, are used to initiate three different segments of the image histogram. The partial likelihoods of the three segments start from zero, \(l_{1.0}=l_{2.0}=l_{3.0}=0\) and grow with the same slope (lambda) which increases monotonically. Each segment of the image histogram is defined by a lower and an upper threshold, which are moving towards lower and higher pixel values respectively as the segment grows driven by the increasing slope of its partial likelihood. The inner thresholds meet at 192.0 (for \(\lambda=4.8\)) and at 69.5 (for \(\lambda=6.9\)) as depicted in Fig. 2.b, where the likelihood slopes vs. the levels of the moving thresholds are illustrated. The partial likelihoods corresponding to the three segments of the histogram are estimated from the standard deviations (s.t.d.) of the pixel values within each segment. Fig. 3 presents the evolution of the corresponding standard deviations (\(\sigma_{1,2,3}\)), the partial likelihood values and their slopes for the three segments. Segmentation of the original image using the obtained thresholds is given in Fig. 4.

The Lagrange multipliers, denoted as \(\lambda_{p,t}\), have to increase monotonically during growth. It turns out that a careful selection of the seeds that initiate the segmentation algorithm yields region-growing according to such a criterion in most practical applications of the proposed method. This is illustrated in Figs. 3.

Should Shannon’s Entropy be used in order to determine the thresholds of the image in Fig. 1 according to the theory in Section 2, the estimated thresholds are, in order of importance: 75, 213, 135 and other gray-level values that lead to less significant reductions of the Shannon’s energy as determined by the local minima in Fig. 5. Standard \(S\)-functions (with \(b_0=24\)) are used to derive the membership functions. These threshold values compare well against our results using the proposed segmentation algorithm.

5. DISCUSSION

The proposed algorithm may be applied slightly modified in the context of conventional region-growing. The partial likelihood functions are defined upon growing regions of the image per se instead of adaptive intervals of its histogram. Maintaining monotonicity of the likelihood slopes in such a case turns out to be a tricky task. Thus application of proper splitting and merging rules during the execution of a generalization of the proposed algorithm becomes imperative for a proper segmentation.

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6. REFERENCES


FIGURES

Fig. 1 : Original image (the proposed algorithm is applied to its global histogram)

Fig. 2 : Upper part: Histogram & initial peaks – Lower part: Slopes of the partial likelihoods ($\lambda_{k,i} \leq \lambda_{k,i+1}$)

Fig. 4 : Segmented image (thresholds at 69.5 and 192.5)

Fig. 5 : Local minima determine acceptable thresholds according to the minimization of Shannon’s entropy

Fig. 3.a : Standard deviations of the pixel values during segmentation for the three different histogram regions