Power limited Cooperative Diversity in Rayleigh Fading for Wireless Ad-hoc Networks

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Abstract—The performance of a cooperative diversity with selection combining is analyzed in Rayleigh fading channels. The effect of limited power at the relay node is also presented. Our results show that when the average signal-to-noise ratio (SNR) of each branch is identical, the single relay cooperative diversity with selection combining provides 13.5 dB power gain over the non-cooperative system at the outage probability of $1 \times 10^{-3}$. We also show that the power limitation on the relay node degrades the performance of the cooperative diversity significantly. Based on our results, it is concluded that the use of a relay node with limited power should be prevented in cooperative diversity systems.

Keywords: cooperative diversity, relay network, selection combining.

I. Introduction

The mobile wireless channel suffers from fading, meaning that the signal attenuation can vary significantly over the course of a given transmission. Recently, relay transmission and cooperative diversity have been focused to mitigate the effects of shadow fading and to reduce power consumption of wireless ad-hoc networks [1]-[4]. In [1], the outage probability with two-hop relayed transmission is derived for decode and forward (regenerative) and amplify and forward (non-regenerative) systems in Rayleigh fading channels. More recently, a cooperative diversity with maximal-ratio combining (MRC) which coherently combines the signals from a source node and a relay node is considered to improve the performance of the relayed network in fading environments [4].

It is well known that the MRC receiver yields the best diversity combining performance in wireless fading channels when compared to other diversity combining techniques. However, it requires the perfect knowledge of the channel state information (CSI) [5], [6]. In addition, the performance of the MRC receiver is very sensitive to imperfect CSI. In general, the tracking of rapidly changing CSI in Rayleigh fading increases the receiver complexity, which consequently causes more power consumption at the receiver. Since wireless ad-hoc sensor networks are power limited, the increase of power consumption of each node is very fatal to the network lifetime and reliability [7], [8].

In this paper, we consider a cooperative communication system in Rayleigh fading channels. We first derive the performance of the cooperative diversity when the selection combining is employed at the destination receiver. The selection diversity combining, which selects the strongest signal among receiving branches, hence, does not need CSI for combining. However, tracking the channel fading stringently for CSI is crucial to MRC. Therefore, the receiver with selection combining can have relatively simple structure for its implementation [6], [9], [10]. We also examine the effect of the limited transmit power at a relay node on the performance of the cooperative diversity system.

This paper is organized as follows. Section II provides background regarding the considered system model. The outage probability of the cooperative selection diversity is derived as a performance measure in Section III. The effect of power limitation at the relay node is discussed in Section IV. Section V presents some numerical examples and discussions. Finally, Section VI summarizes the results of this paper.

II. System Model of a Cooperative Diversity

![Fig. 1. System model of considered cooperative diversity.](image-url)

The system model of a cooperative diversity is shown in Fig. 1. S, R and D denote source node, relay node, and destination node, respectively. In this work we consider...
single relay node for simplicity. Generally, there are two types of cooperative networks - decode and forward (DF) and amplify and forward (AF) cooperative networks. The relay node of DF networks decodes the received signal from the source node and forwards the decoded signal to the destination node. While that of the AF cooperative networks amplify and forwards the received signal to the destination node. Since a wireless ad-hoc network is power-limited, the AF cooperative diversity is preferred for the less power consumption than DF diversity [11]. In this paper, we consider the AF cooperative diversity and assume each wireless channel between nodes has independent Rayleigh fading.

The source node transmits during the first time slot and the relay node transmits during the second time slot. The signals from the source node and from the relay node are selectively combined at the destination. Let us define the signal path from the source node to the destination node is direct path, and the signal path from the relay node to the destination node is relay path. Then, the received signal at the destination node during the first time slot is given by

\[ y_D = \sqrt{P \sigma_D^2} x + n_D \]  

where \( P \) denotes the channel gain between the source node and the destination node. \( P \) is the transmitted power at the source node and \( x \) is a binary information symbol. \( n_D \) denotes additive white noise, which has zero mean and complex Gaussian distributed with variance \( N_0 \).

The received signal at the destination node during the second time slot can be written by

\[ y_R = \sqrt{P_R \sigma_R^2} u + n_D \]

where \( P_R \) is the transmitted power at the relay node, and \( \sigma_R^2 \) denotes the channel gain between the relay node and the destination node. \( u \) equals \( \sqrt{P_R \sigma_R^2} x + n_R \) where \( \sigma_R \) denotes the channel gain between the source node and the relay node, and \( n_R \) denotes additive white noise having zero mean and complex Gaussian distributed with variance \( N_0 \).

### III. Selection Diversity Combining

As mentioned earlier, the MRC scheme requires co-phasing and weighting blocks in the receiver, which leads to implementation complexity and processing power consumption. In order to avoid these blocks, we consider selection combining cooperative diversity in the paper.

Selection combining scheme selects the strongest signal among the received signals. When all of the received SNR from the \( n \) branches are simultaneously less than or equal to a given threshold SNR \( \Gamma \), then the outage is declared. Then, the outage probability \( P_{out}(\Gamma) \) can be written by

\[ P_{out}(\Gamma) = \Pr(\gamma_1, \gamma_2, ..., \gamma_n \leq \Gamma) \]  

where \( \gamma_i (1 \leq i \leq n) \) is the instantaneous SNR of each branch. Assuming the received SNR of each branch is independent, the outage probability is

\[ P_{out}(\Gamma) = \Pr(\gamma_1 \leq \Gamma) \Pr(\gamma_2 \leq \Gamma) ... \Pr(\gamma_n \leq \Gamma). \]

As shown in Fig.1, there are two receiving signals at the destination node in our cooperative communications model, signals from the direct path during the first time slot and from the relay path during the second time slot. The outage probability \( P_{out}(\Gamma) \) of the direct path in Rayleigh fading is given by [11]

\[ P_{D, out}(\Gamma) = \Pr(\gamma_D \leq \Gamma) = 1 - \exp(-\Gamma/\bar{\gamma}_D) \]  

where \( \gamma_D \) and \( \bar{\gamma}_D \) are the instantaneous received SNR and the average received SNR of the direct path, respectively. The outage probability \( P_{R, out}(\Gamma) \) on the relay path is given by [4], [8]

\[ P_{R, out}(\Gamma) = \Pr(\gamma_R \leq \Gamma) = 1 - \frac{2\Gamma}{\sqrt{\bar{\gamma}_{SR} \bar{\gamma}_{RD}}} K_1\left(\frac{2\Gamma}{\sqrt{\bar{\gamma}_{SR} \bar{\gamma}_{RD}}}\right) \exp\left(-\Gamma/\bar{\gamma}_{SR} \bar{\gamma}_{RD} \right) \]

where \( \gamma_R \) is the instantaneous received SNR of the relay path. \( \bar{\gamma}_{SR} \) denotes the average received SNR from the source node to the relay node, and \( \bar{\gamma}_{RD} \) denotes the average received SNR from the relay node to the destination node. \( K_1 \) is the 1st order modified Bessel function of the 2nd kind.

Since we assume the received signals from each branches are independent, the outage probability with the selection combining can be given by

\[ P_{out}(\Gamma) = P_{D, out} P_{R, out}. \]

The outage probability of MRC in cooperative system under Rayleigh fading is [12], [13]

\[ P_{out}(\Gamma) = \frac{\Gamma}{2\sigma_{SD}^2} \left( \frac{1}{\sigma_{SR}^2} + \frac{1}{\alpha \sigma_{RD}^2} \right) \left( \frac{N_0}{P_S} \right)^2 \]

where \( \alpha \) is given by \( P_R / P_S \).
IV. Effect of Power Limit

The performance of cooperative diversity system can be improved under the assumption of the sufficient transmit power at each node. In ad-hoc networks, however, the transmit power of a node can be different and limited, indicating that the maximal available transmit power is restricted. In this section, we assume the transmit power at the relay node is limited. Let us define the power limitation coefficient $\beta$ as

$$\beta = \frac{\bar{T}_{RD,\text{max}}}{\bar{T}_{SR}}. \quad (9)$$

Then, the outage probability $P_{out}(\Gamma)$ of the relay path can be calculated as

$$P_{out}(\Gamma) = 1 - \frac{2\Gamma}{\bar{T}_{SR}\sqrt{\beta}} K_{1}\left(\frac{2\Gamma}{\bar{T}_{SR}\sqrt{\beta}}\right) \exp\left[-\Gamma(1 + \beta) / (\beta\bar{T}_{SR})\right]. \quad (10)$$

After some manipulation [12], for high SNR, the outage probability in (8) can be written by

$$P_{out}(\Gamma) \approx \frac{\Gamma^2}{2\bar{T}_{SD}} \left(1 - \frac{1 - \beta}{\bar{T}_{SR}}\right). \quad (11)$$

V. Numerical Results and Discussions

![Fig. 2. Outage probability of cooperative diversity with selection combining.](image)

Fig. 2. Outage probability of cooperative diversity with selection combining.

For numerical examples of the no power limited case, we assume the received average SNR from the source-relay path $\bar{T}_{SR}$ is identical with that of the relay (relay-destination) path $\bar{T}_{RD}$. We further define $\bar{T}_R = \bar{T}_{SR}$.

= $\bar{T}_{RD}$. Fig. 2 depicts the performance of the cooperative diversity with selection combining (no power limited case). $\bar{T}_{SD}$ denotes the received average SNR at the destination node on the direct (source-destination) path. It is found that the performance gain provided by the cooperative diversity over the non-cooperative diversity scheme equals 13.5 dB for $\bar{T}_{SD} = \bar{T}_{RD}$, 15 dB for $\bar{T}_{SD} = 0.5\bar{T}_{RD}$, and 17 dB for $\bar{T}_{SD} = 0.2\bar{T}_{RD}$ at the outage probability of $1 \times 10^{-3}$. We note that the performance gain is more significant as the received SNR from the direct path is reduced.

Fig. 3 shows the outage probability versus power limitation coefficient of a selection combining diversity system (when $\bar{T}_{SD} = \bar{T}_{SR}$). It is noticed that the performance of the cooperative diversity with selection combining is more sensitive to the power limitation factor than that with the MRC. When $\bar{T}_{SR}/\Gamma = 20\, \text{dB}$ and $\beta$ is less than -25 dB, it means the transmit power of the relay node is limited, the outage probability degrades around $1 \times 10^{-2}$. When $\beta$ increases from -20 dB to 0 dB, the performance of SC improves. $\beta$ is greater than 5 dB, however, the performance improvement is negligible. On the other hand the performance of MRC is continuously improved with $\beta$.

VI. Conclusions

In this work we have analyzed the performance of a selection combining cooperative diversity in Rayleigh fading channels. It was shown that when the received average SNR on the source-relay path, the relay-destination path, and the source-destination path are identical, the performance gain of the selection combining cooperative diversity is 13.5 dB over the non-cooperative system at the outage probability of $1 \times 10^{-3}$. We have also
investigated the effect of limited power on the relay path by deriving the outage probability in terms of the power limitation coefficient. Since the system performance of the cooperative diversity with selection combining is vulnerable to the power limitation on the relay path, the use of insufficient transmit power node is not recommended as a relay node in cooperative diversity systems.

References