Abstract - Super-orthogonal space-time trellis codes (SOSTTC) are full rate, full diversity space-time codes with high coding gains for quasi-static fading channels. In this paper, this design approach is extended to fast fading channels for the case of 4 transmit antennas and a new SOSTTC is proposed for BPSK modulation based on the design criteria valid for this type of channels. The new code has 16-state trellis for BPSK to avoid parallel transitions which restrict the error performance. The frame error performances are presented showing that the proposed code have superior performance in fast fading case compared to the reference codes.

I. INTRODUCTION

A joint design of coding, modulation, transmit and receive diversity was proposed by Tarokh et al [1] resulting an improvement in the error performance of wireless systems. This approach is called space-time trellis coding. It has been shown that such codes provide full diversity gain as well as additional signal-to-noise ratio (SNR) advantage, called the coding gain. They also derived an analytical bound for the pairwise error probability and obtained design criteria to improve the space-time code error performance and proposed code design rules for achieving full diversity for two transmit antennas. Using these design rules, examples of codes with full diversity are constructed in [1] but coding gains not necessarily optimized.

A simple code providing full diversity for two transmit antennas is introduced by Alamouti in [3] where transmitted symbols can be separately decoded based on a linear processing at the receiver. In [4], Alamouti’s scheme is generalized to an arbitrary number of transmit antennas and named as space-time block coding. When more than two transmit antennas are considered, a full rate and full diversity orthogonal space-time block code (STBC) design is not possible. A STBC provides full diversity and a very simple decoding scheme, but despite the name, its main goal is not to provide additional coding gain [4]. This is in contrast to space-time trellis codes (STTC) that provide full diversity as well as significant coding gains but at a cost of higher decoding complexity. Several schemes based on the concatenation of the STBC with an outer trellis code are then proposed in the literature [6], [7] to achieve satisfactory coding gains. The main disadvantage of these schemes is their rates which are below the possible maximum rate.

A new class of space-time codes called super-orthogonal space-time trellis codes (SOSTTCs) is introduced in [2] to overcome this problem. These codes combine set-partitioning and a super set of orthogonal STBCs in a systematic way to provide full diversity and improved coding gain over earlier STTC constructions. The super-orthogonal signal set is obtained by a constellation rotation. The structure of the new codes allows an increase in the coding gain, while providing full diversity and full rate. Since orthogonal designs have been used as building blocks in SOSTTCs, the decoding complexity remains low. In addition to Jafarkhani and Seshadri’s work [2], independent methods to expand the orthogonal matrix set were developed in [9] and [10].

In this paper, SOSTTC design technique is applied to space-time BPSK modulation schemes for fast fading channels in the case of 4 transmit antennas using the appropriate real orthogonal design from [4] and using the design criteria valid for this type of channel. Avoiding parallel state transitions in its trellis, a 16-state SOSTTC is proposed for BPSK modulation with maximized symbolwise Hamming distance and maximized sum-product distance.

In Section II, we provide space-time code design criteria for fast Rayleigh fading channels and give a brief review of SOSTTC design technique. The new BPSK SOSTTC designed for fast fading channels is presented in Section III. We present the frame error performance results obtained by computer simulations.
in Section IV and concluding remarks are provided in Section V.

II. CODE DESIGN CRITERIA

In the evaluation of space-time code performance in quasi-static fading channels where the fading coefficients remain constant over one symbol frame and vary from one frame to the next, the critical parameters are the minimum rank and the minimum determinant of the difference matrix between all possible transmitted and erroneously decided symbol sequences. However for fast fading channel case, where the fading coefficients remain constant over one symbol interval and vary from one interval to the next the important parameters are the symbol-wise Hamming distance and the sum-product distance.

In general, considering a system of \( n \) transmit and \( m \) receive antennas, the transmitted and erroneously decided symbol sequences can be written as

\[
e = c_1^1 c_1^2 \ldots c_1^n c_2^1 \ldots c_2^n \ldots c_l^1 \ldots c_l^n
\]

and

\[
e = e_1^1 e_1^2 \ldots e_1^n e_2^1 \ldots e_2^n \ldots e_l^1 \ldots e_l^n
\]

respectively, where \( l \) denotes the time instances.

Assuming complex channel gains have independently Rayleigh distributed absolute values, the pairwise error probability is well approximated by [1]

\[
P(e \rightarrow e) \leq \prod_{t \in \eta(c,e)} \left( |e_t - e_t| \right)^2 E_s \end{array}^{\frac{m}{4N_0}}
\]

where \( E_s \) is the symbol energy, \( N_0/2 \) is the variance per dimension of the zero-mean complex Gaussian noise samples and \( \eta(c,e) \) is the set of time instances that \( c \) and \( e \) differ. The number of elements in \( \eta(c,e) \), denoted as \( l_\eta \), is defined as the number of space-time symbols in which the two sequences differ, also called the space-time symbolwise Hamming distance. Since

\[
|e_t - e_t| = \sum_{i=1}^{n} |c_i^t - e_i^t|^2
\]

the product of distance sums along error event path pair is equal to

\[
\prod_{t \in \eta(c,e)} |e_t - e_t|^2
\]

which is also called the sum-product distance. It is seen from (1) that the minimum value of \( l_\eta \) and the minimum value of the product of distance sums dominates the error event probability. The code design criteria for fast fading channels can be summarized as follows:

\( a) \) Maximize the minimum space-time symbolwise Hamming distance between all pairs of distinct codewords.

\( b) \) Maximize the minimum sum-product distance along the path pair(s) with minimum symbolwise Hamming distance.

The design criteria \((a)\) and \((b)\) correspond to the diversity and coding gain maximizations, respectively.

In SOSTTCs, a STBC with specific constellation symbols is assigned to transitions emanating from a state. In general, for a \( kn \) STBC, choosing a trellis branch emanating from a state is equivalent to transmitting \( kn \) symbols from \( n \) transmit antennas in \( k \) time intervals. For two transmit antennas the class of orthogonal transmission matrices is given as,

\[
C(x_1, x_2, \theta) = \begin{pmatrix}
x_1 e^{j\theta} & x_2 \\
-x_2 e^{j\theta} & x_1^*
\end{pmatrix}
\]

where \( x_1 e^{j\theta} \) and \( x_2 \) are the transmitted symbols from the first and second transmit antennas, at the first symbol interval, and the symbols \(-x_2 e^{j\theta} \) and \( x_1^* \) are transmitted from the first and second transmit antennas, at the second symbol interval, respectively. The parameter \( \theta \) is chosen such that for any \( x_1 \) and \( x_2 \) from the original constellation points, the rotated signals are also from the same constellation. \( \theta = 0, \pi \) for BPSK and \( \theta = 0, \pi \) or \( \theta = 0, \pi/2, 3\pi/2 \) for QPSK can be used without expanding the signal constellation. Super-orthogonal codes expand the number of available orthogonal matrices for different values of parameter \( \theta \) to provide the necessary redundancy to design full rate, full diversity trellis codes. For fast fading channels, set partitioning can be performed as in the quasi-static fading channel case but the maximization of the sum-product distance is considered instead of the coding gain distance.

Full-rate orthogonal designs with complex symbols are impossible for more than two transmit antennas whereas a full-rate \( N \times N \) real orthogonal design only exists for \( N = 2, 4, 8 \) [4]. The only example of a full-rate full-diversity complex space-time block code using orthogonal designs is the Alamouti’s scheme [3] which is used as a starting point to obtain (4). An example of a \( 4 \times 4 \) real orthogonal design from [4] is given as follows:

\[
C(x_1, x_2, x_3, x_4) = \begin{pmatrix}
x_1 & x_2 & x_3 & x_4 \\
-x_3 & -x_2 & x_1 & -x_4 \\
-x_4 & -x_3 & x_2 & x_1
\end{pmatrix}
\]

Rows of this matrix denote time instances whereas the columns denote transmit antennas. For example elements of the first row \( x_1, x_2, x_3, x_4 \) are the transmitted symbols from transmit antennas one to four, at the first symbol interval, respectively.

Since we aimed to design BPSK SOSTTC for 4 transmit antennas and maintain orthogonality, the
design given in (5) is suitable to assign to transitions emanating from a state. To expand the number of orthogonal matrices, similar to the case of two antennas in (4), phase rotations can be used as follows:

\[
C(x_1, x_2, x_3, x_4, \theta_1, \theta_2, \theta_3, \theta_4) = \begin{pmatrix}
x_1e^{j0_1} & x_2e^{j0_2} & x_3e^{j0_3} & x_4 \\
-x_1e^{j\theta_1} & x_2e^{j\theta_2} & -x_3e^{j\theta_3} & x_4 \\
-x_1e^{j\theta_1} & x_2e^{j\theta_2} & x_3e^{j\theta_3} & -x_4 \\
-x_1e^{j\theta_1} & -x_2e^{j\theta_2} & x_3e^{j\theta_3} & x_4 \\
\end{pmatrix}
\] (6)

Since the constellation is real and an expansion in the constellation signals is not desired, \( \theta_i = 0, \pi \) where \( i = 1, 2, 3 \) can be used, resulting a potential sign change for each column. In general, for \( N \) transmit antennas, \( N-1 \) rotations can be used [2].

III. CODE DESIGN

In this section, a BPSK SOSTTC is designed for fast fading channels when four transmit and arbitrary number of receive antennas are considered. To maximize the minimum space-time symbolwise Hamming distance between all pairs of distinct transmission matrix sequences, parallel transitions between any state pair are avoided. If two sequences diverge from a common state to different states they have to go at least two transitions before they merge to a common state. Therefore the shortest error event path will have two steps and the minimum value of space-time symbolwise Hamming distance would be increased.

Regarding four transmit antennas and avoiding parallel transitions, a trellis of at least 16-state should be considered for full rate BPSK SOSTTC. The full diversity criterion is satisfied for every path pair due to the orthogonality of the matrix given in (6). To fulfill the second design criterion, a computer search program was developed and the transmission matrices were assigned to the trellis branches so that the minimum sum-product distance over all possible two-step transmission matrix sequence pairs was maximized. Considering the orthogonal design in (6), for different values of \( \theta_i, i=1,2,3 \) which are \( (\theta_1, \theta_2, \theta_3) = \{(0,0,0), (0,0,\pi), (0,\pi,0), (0,\pi,\pi), (\pi,0,0), (\pi,0,\pi), (\pi,\pi,0), (\pi,\pi,\pi)\} \) there are 8 possible orthogonal sets that can be used. The computer search was effectuated for codes with all combinations of possible sets. Best result is obtained using the sets \((0,0,0)\) and \((\pi,\pi,\pi)\). To express the trellis more clearly and to diminish the complexity in visual aspect, the orthogonal matrices are denoted by \( C^j \)

where \( i=1,2 \) represents \((\theta_1, \theta_2, \theta_3) = (0,0,0)\) and \((\theta_1, \theta_2, \theta_3) = (\pi,\pi,\pi)\), respectively. On the other hand \( j=1,2,\ldots,16 \) denotes all realizations of the binary codeword \( x_1, x_2, x_3, x_4 \) as 0000, 1111, 0011, 1100, 0101, 1010, 0110, 1001, 0001, 1110, 0010, 1101, 0100, 1011, 1111, 1000, respectively, which are mapped to the BPSK symbols by the rule \( 0 \rightarrow -1, 1 \rightarrow 1 \). The proposed 16-state SOSTTC for BPSK modulation is given in Fig. 1 where transmission matrices assigned to branches originating from each state are given on the left side of the trellis. The space-time symbolwise Hamming and sum-product distance values for this code are 8 and 32, respectively.

IV. SIMULATION RESULTS

We evaluated the frame error probability (FER) of the proposed 16-state BPSK SOSTTC by computer simulations. In the simulations, a MIMO system with four transmit and one receive antenna was assumed. A Viterbi decoder which operates on the code trellis is employed at the receiver. It was assumed that the receiver has perfect knowledge of the channel fading coefficients. It was also assumed that the signals received from different transmit antennas experience independent fading, which means that the fading coefficients are independent complex Gaussian random variables with mean zero and variance \( \frac{1}{2} \) per dimension. In all simulations, a frame consisting of 130 transmitted symbols from each transmit antenna was considered and the FER of the proposed BPSK code using four transmit antennas and one receive antenna was obtained. The fading coefficients were assumed to be constant over four consecutive time intervals and independently vary from four time intervals to the next four, which is equivalent to assume that a perfect interleaver is included to the system.

For the case of 4 transmit antennas, any BPSK SOSTTC, designed according to fast fading channel criteria is not available in the literature. Performance comparison of the proposed code is made with the fast fading channel performance of the 2-state BPSK SOSTTC given in [2] which is designed according to
quasi-static fading channel criteria for four transmit antennas. FER performance of the 4-state super-orthogonal space-time BPSK trellis code [11] designed for two transmit antennas regarding fast fading channel criteria is also considered as reference code. Fig. 2 shows the FER results versus signal-to-noise ratio (SNR) at rate $r = 1$ bits/sec/Hz for the 16-state code given in Fig.1, denoted by “16-state NEW”. The FER performance of the reference 2-state SOSTTC in fast fading channels denoted “2-state SOSTTC” is also included in Fig. 2. Since the “16-state NEW” code is designed according to fast fading channel criteria, it outperforms the reference 2-state code approximately by 4 dB at a FER of $10^{-3}$. As can be seen from Fig.2 an increase in diversity results a steeper performance curve and also an increase in the number of transmit antennas and in the number of states results in a considerable amount of coding gain as expected.

![Graph showing FER performances of 16-state BPSK SOSTTC and reference codes on Rayleigh fast fading channels](image)

V. CONCLUSION

In this paper, a new super-orthogonal space-time BPSK trellis code designed for four transmit antennas in fast fading channels have been proposed. The new code provides full rate, full diversity, and high coding gain. Simulation results confirm that the proposed code offer a better performance compared to their counterparts given in the literature. The research is restricted to BPSK scheme, since full-rate complex orthogonal designs for four transmit antennas does not exist. Allowing a decrease in rate or using quasi-orthogonal transmission matrices, the research can be expanded to complex constellation schemes.

REFERENCES


