A Blind Multiuser Detector Based on Lagrange Neural Network*

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Abstract-In this paper, a blind multiuser detector based on Lagrange neural network (LNN) is proposed. Based on the minimum output energy (MOE) criterion, the blind detection algorithm is formulated as a constrained optimization problem inherently and then resolved efficiently using neural network. It is shown that compared with previous MOE blind detector, the proposed LNN blind multiuser detector offers better performance, lower computational complexity, faster convergence speed and real-time operation.

I. INTRODUCTION

Code division multiple access (CDMA) implemented with direct sequence (DS) spread spectrum signaling is a technology applied to a number of important applications nowadays such as mobile telephony, wireless networks and personal communications. In these years, multiple access interference (MAI) becomes an intrinsic factor for severe performance degradation that necessitates the application of signal processing techniques to improve quality. Multiuser detection schemes developed over the last years successfully mitigate MAI achieving at the same time significant capacity improvement for the corresponding CDMA systems. Due to their capability to combat MAI, multiuser detection schemes, especially blind multiuser detection algorithms, have attracted considerable attention and currently significant research is devoted in this direction. Honing [1] proposed a linear blind multiuser detector to minimize the output energy subject to one or more constraints that ensure the output gain to the desired user is held constant. Poor [2] proposed a constrained RLS blind multiuser detector based on MOE criterion. But the classical blind multiuser detection algorithms are based on the descendant gradient technique. It needs a certain number of bit times to reach the optimum solution.

In this paper, the motivation is to reach the optimum solution in the blind detection algorithm by using neural networks fast parallel processing capabilities. It accelerates the convergence process of the adaptive filter coefficients. The rest of this paper is organized as follows. Sec.II presents the signal model and the MOE blind multiuser detector. Sec.III provides briefly some background on LNN. Sec.IV describes the proposed neural network approach. Sec.V shows the simulation results. Finally, our conclusions are discussed.

II. MOE BLIND MULTIUSER DETECTOR

A. Signal Model

Consider a K-user synchronous DS/CDMA communication system through an additive white Gaussian noise (AWGN) channel. The received continuous time baseband signal can be modeled as

\[ y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t) \]  

where \( s_k(t) \) is signature waveform of the \( k \) th user, \( A_k \) is the received amplitude of the \( k \) th user, \( b_k \in \{ \pm 1 \} \) is the symbol of the \( k \) th user, \( n(t) \) is a white Gaussian noise with zero mean and unit variance. \( \sigma^2 \) denotes the power of the AWGN.

The received signal is sampled at the chip rate. Using matrix notation, (1) can be transformed into

\[ y = S A b + \sigma n \]  

where \( y = [y_1, y_2, \ldots, y_N]^T \), \( S = [s_1, s_2, \ldots, s_K] \), \( A = \text{diag}[A_1, A_2, \ldots, A_K] \), \( b = [b_1, b_2, \ldots, b_K]^T \), \( n \) is white Gaussian noise vector. \( N \) is the spreading factor of all signature waveforms.

B. MOE Blind Multiuser Detector

In [1] a linear detector based on the minimum mean square error (MMSE) detector is derived. The desired user is taken to be user \( 1 (k = 1) \). A linear detector estimates the user's transmitted bit by taking the sign of the inner product of \( y \) with a properly selected vector (filter) \( c_k \) of length \( N \), specifically \( \hat{c}_k = \text{sgn}(c^T_k \cdot y) \). Then MOE is then defined as

\[ \text{MOE}(c_k) = \mathbb{E} \left[ (c_k^T \cdot y)^2 \right] \]  

So the blind detection can be transformed into minimizing the MOE, that is

\[ c_{\text{opt}} = \arg \min \mathbb{E} \left[ (c^T \cdot y)^2 \right] \]  

s.t. \( s^T \cdot c = 1 \)

where \( c_{\text{opt}} \) denotes the optimum weights of the detector, and \( s = s_1, c = c_1 \).

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III. LAGRANGE NEURAL NETWORK

Among the many types of neural networks, a Lagrange neural network [4] has been selected in this paper to obtain the solution of the $c_{opt}$ in (4).

Consider the optimum problem

$$\begin{align*}
\min & \quad f(x) \\
\text{s.t.} & \quad g(x) = 0
\end{align*}$$

(5)

where $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$, $f(x)$ is a scalar function, $g(x) = [g_1(x), g_2(x), \ldots, g_m(x)]^T$ is a $m$ dimension vector function, $m \leq n$. Assume $f(x)$, $g(x)$ are continuous, and their second order derivatives exist and are continuous.

The Lagrange function of (5) is defined as

$$L(x, \lambda) = f(x) + \lambda^T g(x)$$

(6)

where $\lambda^T = (\lambda_1, \lambda_2, \ldots, \lambda_m)^T \in \mathbb{R}^m$, $\lambda_i$ is Lagrange multiplier.

According to the theory of optimization, the necessary condition of solving (5) is

$$\begin{align*}
\frac{\partial L(x, \lambda)}{\partial x} &= \frac{\partial f(x)}{\partial x} + \lambda^T \frac{\partial g(x)}{\partial x} = 0 \\
\frac{\partial L(x, \lambda)}{\partial \lambda} &= g(x) = 0
\end{align*}$$

(7)

(7) can be written as

$$\begin{cases}
\frac{\partial f(x)}{\partial x} + \sum_{j=1}^{m} \lambda_j \frac{\partial g_j(x)}{\partial x_j} = 0 & i = 1, 2, \ldots, n \\
g_j(x) = 0 & j = 1, 2, \ldots, m
\end{cases}$$

(8)

Using above Lagrange multipliers, we construct a Lagrange neural network (LNN) to solve (5). Its equilibrium point satisfies (7) or (8). The dynamic of this neural network is determined by below equations

$$\begin{align*}
\frac{dx}{dt} &= -\frac{\partial L(x, \lambda)}{\partial x} = -\frac{\partial f(x)}{\partial x} - \lambda^T \frac{\partial g(x)}{\partial x} \\
\frac{d\lambda}{dt} &= \frac{\partial L(x, \lambda)}{\partial \lambda} = g(x)
\end{align*}$$

(9)

(9) can be written as

$$\begin{cases}
\frac{dx_i}{dt} = -\frac{\partial f(x)}{\partial x_i} - \sum_{j=1}^{m} \lambda_j \frac{\partial g_j(x)}{\partial x_j} & i = 1, 2, \ldots, n \\
\frac{d\lambda_j}{dt} = g_j(x) & j = 1, 2, \ldots, m
\end{cases}$$

(10)

where $x_i, \lambda_j$ are the state variables of the variable neurons and constraint neurons respectively. From the dynamic behavior of neural network (10), we can see it can be realized by circuit directly [3].

From (10), we also can see along the trajectory of the neural network, the Lagrange function always decreases with $x$ and increases with $\lambda$. When the neural network evolves to stabilization, the equilibrium point of the neural network is the optimum solution of (5). [4] has proved the stability of this neural network.

IV. LNN BLIND MULTIUSER DETECTOR

Comparing (4) and (5), we can find they are similar in the forms. The solution of (4) can be obtained by above neural network. For practical implementation of neural algorithm on mean energy function, we can simplify (4) in

$$c_{opt} = \arg\min_{c} E[(c^T \cdot y)^2]$$

$$= \arg\min_{c} \frac{1}{P} \sum_{i=1}^{P} [(c^T \cdot y(i))^2]$$

$$= \arg\min_{c} \frac{1}{2} c^T Rc$$

$$s.t. \quad s^T \cdot c = 1$$

(11a)

where

$$R = \sum_{i=1}^{P} y(i) \cdot y^T (i),$$

(11b)

$P$ is the averaging window in bit time. The algorithm adopts weighted accumulation to replace expectation operation, and the relationship between BER and estimation sample length $p$ is shown in the fig.1. In the computer simulation, SNR of the desired user is 10dB, seven interfering users with 10dB SNR, and two interfering users with 20dB SNR. After the experiment, we can draw such conclusion that when $p = 100$, the detector can obtain the best BER performance.

![Fig.1 BER verse numbers of estimation sample](image)

We define the Lagrange function of the neural network to solve (11) is

$$L(c, \lambda) = \frac{1}{2} c^T Rc + \lambda (s^T \cdot c - 1)$$

(12)

where $\lambda$ is the Lagrange multiplier. According (9),
obtain the dynamic function of LNN_BMUD
\[
\frac{dc}{dt} = -\frac{1}{2} \frac{\partial (c^T R c)}{\partial c} - \lambda \frac{\partial (s^T \cdot c - 1)}{\partial c}
\]
\[
\frac{d\lambda}{dt} = s^T \cdot c - 1
\]  \hspace{1cm} (13)

(13) can be written as
\[
\frac{dc_i}{dt} = -\sum_{k=1}^{N} R_{ik} c_i - \lambda s_i \quad i = 1, 2, \cdots, N
\]
\[
\frac{d\lambda}{dt} = \sum_{j=1}^{N} s_j^T c_j - 1
\]  \hspace{1cm} (14)

The stability of the proposed neural network can be analyzed through a suitable extension of the method discussed in [4][5]. The neural network in (13), where the cost function \( L(c, \lambda) \) is given by (12), has a unique equilibrium point, which is globally asymptotically stable.

The circuit of LNN-BMUD corresponding to (13) or (14) is showed in Fig.1. In Fig.2, the elements of \( R, s, s^T \) are realized by connecting resistance directly, and every element of the matrices is the conductance corresponding to the connecting resistance. The steady outputs of the integrators are the values of the corresponding elements in \( c_{opt} \).

![Fig.2 The circuit of LNN-BMUD.](image)

The principle structure of LNN-BMUD is showed in Fig.3.

![Fig.3 The principle structure of LNN-BMUD.](image)

From above analyses, the computational complexity of neural network detection algorithm is \( O(N) \) and the computational complexity of RLS-MOE detection algorithm proposed in [2] is \( O(N^2) \). Comparing them, the former is improved obviously in computational complexity.

Since the analytic expression form of the error bit rate of LNN-BMUD is very difficult to be derived, its performance has been evaluated by means of computer simulations. The steps of simulation are as follows:

**Step 1:** Count \( \hat{R} \) by \( P \) groups of signal sample sequences, initialize the weight vector \( \hat{R} \) and \( \hat{s} \), and select the initial values of neurons.

**Step 2:** Select one neuron, update its state value according to (14).

**Step 3:** Repeat step 2, until the state values of all neurons are invariant.

**Step 4:** Obtain the output of neural network, and then make detection.

V. SIMULATION RESULTS

We now investigate the performance of LNN-BMUD and compare with RLS-MOE blind detector by computer simulation. We consider a synchronous DS/CDMA system with spreading factor \( N = 31 \) and five users \( (K = 5) \) in AWGN channel. The desired user is user 1. \( P = 100 \), forgetting factor \( \lambda = 0.995 \).

Fig.4 shows the corresponding bit error rate (BER) of the detectors after convergence versus signal-to-noise ratio (SNR). The amplitudes of the other users are \( A_1^2 = A_2^2 = \cdots = A_K^2 = 10A_1^2 \). These results show that LNN-BMUD in terms of BER than the RLS-MOE receiver, especially in the high SNR scenario, and show that the use of neural network in the MOE detector provides a relatively low complexity blind detector that has reasonably performance.

![Fig.4 Bit error rate versus SNR.](image)

Fig.5 shows the steady state signal-to-interference-and-noise ratio (SNR) of the detectors versus number of iterations. The SNR of user 1 is 20dB. There are four 10dB MAIs and one 20dB MAI in the channel, all relative to the desired user's signal. The SIR level of LNN-BMUD is slightly higher RLS-MOE detector. Moreover, the steady-state SIR of LNN_BMUD is closer to theory value, better performance as the increase of \( P \). Comparing with RLS-MOE detector, LNN-BMUD needs
less data to obtain the same SIR. It is shown that its convergence speed is faster. Fig.5 SIR versus number of iterations.

Fig.6 shows the SIR comparison for variable environment at SNR=20dB, the robustness of this detector (the mismatch of the spreading code) was further researched.

![Fig.6 SIR comparison for variable environment at SNR=20dB](image)

In the experiment, the total iteration times are 1000, SNR is 20dB, the initial interfering users are 15 with ten 10dB and five 15dB interfering users. When the number of iterations reaches 300, entering three 15dB interfering users; when iteration times come to 600, removing two 10dB interfering users, meanwhile introducing two 20dB interfering users; while the number of iterations reaches 800, removing three 20dB interfering users from the communication channel. Finally, we come to a conclusion that the traceability of LNN is stronger than that of RLS-MOE algorithm.

Fig.7 shows the corresponding bit error rate (BER) of the detectors after convergence versus signal-to-noise ratio (SNR). These results show that the BER of neural network detector in multipath is close to that of the network receiver in single path, especially in the high SNR scenario.

VI. CONCLUSION

In this paper we have considered a DS-CDMA communication system, in which a blind detector based on neural network is proposed. The simulation results demonstrate that the proposed neural network receiver can be an effective approach to reach fast convergence speeds and clearly outperforms classical approaches. The simulation results demonstrate that the proposed neural network receiver can be an effective approach to reach fast convergence speeds and clearly outperforms classical approaches and conclude that as flows:

With the SNR is 10dB, the BER of LNN blind multi-user detection in multiple paths is 0.01 larger than that in single path; while if SNR is larger than 13 dB, the BER performance of LNN blind multi-user detection in multi-path channel is close to that in single path channel. The LNN blind multi-user detection is suitable for multi-path fading channel, compared with RLS blind multi-user detector that is suitable for multi-path channel, the LNN-BMUD has better multi-user detection performances.

Comparing with RLS-MOE detector, LNN-BMUD needs less data to obtain the same SIR. It is shown that its convergence speed is faster and robust.

REFERENCES